EE391D Advanced Topics in Photonics

Waveguide theory
lesson 8

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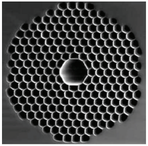
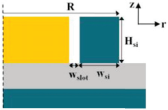
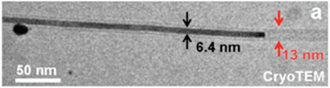
Outline

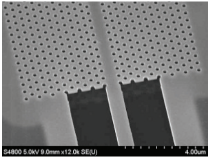
Optical waveguides

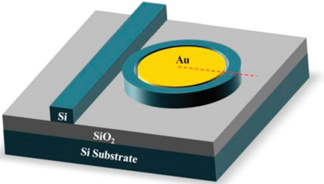
Optical waveguides are photonic structures able to guide light energy inside them. Waveguides are characterized by a geometry that is symmetric along a direction in space, which identifies the propagation direction of energy inside the structure. Optical waveguides can be made of dielectric, metals or a combination of them.

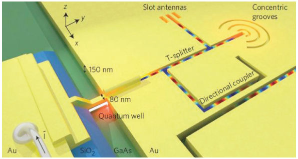












Outline

General theory of waveguide modes

A waveguide mode is a propagating wave solution inside a waveguide structure. A waveguide mode has the following form:

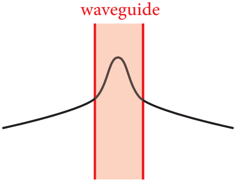
$$\begin{bmatrix} \mathbf{e}(\mathbf{r}, t) \\ \mathbf{h}(\mathbf{r}, t) \end{bmatrix} = \begin{bmatrix} \mathbf{E}_\nu(x, y) \\ \mathbf{H}_\nu(x, y) \end{bmatrix} e^{i\omega t - i\beta_\nu z}, \tag{1}$$

characterized by vectors $\mathbf{E}_\nu$, $\mathbf{H}_\nu$, which describes the spatial distribution of the mode in the plane (x, y) orthogonal to the direction of propagation z, and the propagation constant β_ν. The index ν, which identify the particular mode, can be discrete or continuous, typically depending on the behavior of the mode at $x \to \infty$ and $y \to \infty$. The intensity of the mode is constant along the propagation direction z.

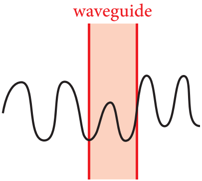
General theory of waveguide modes

For general waveguide structures, there are two different types of modes:

- Guided modes. These modes are characterized by a transverse amplitude that tends to zero outside the waveguide: $\mathbf{E}_\nu(x, y) \to 0$ and $\mathbf{H}_\nu(x, y) \to 0$ for $(x, y) \to \infty$. The mode intensity is localized inside the waveguide, allowing for a guided propagation of energy inside the structure.
- Radiation modes. For this class of modes, the transverse amplitude periodically oscillates outside the waveguide. The mode intensity is localized outside the waveguide.



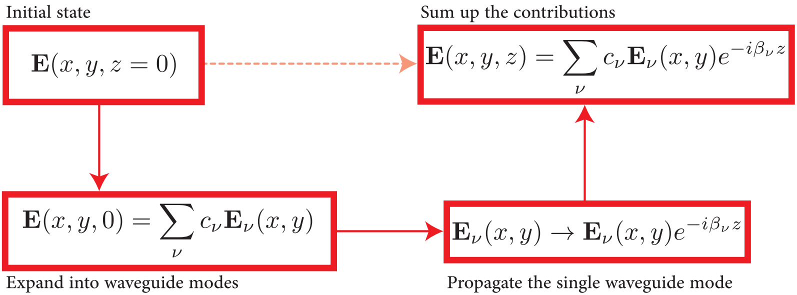
a guided mode



a radiation mode

The concept of waveguide mode

The complete set of modes of a waveguide (guided + radiation) identifies all the possible propagating waves supported by the structure. Any other solution of the system can be expressed as a combination of modes.



Outline

Mode normalization and orthogonality

From Maxwell equations:

$$\begin{cases} \nabla \times \mathbf{E} = -i\omega\mu\mathbf{H}, \\ \nabla \times \mathbf{H} = i\omega\epsilon(\omega)\mathbf{E}, \end{cases} \tag{2}$$

if $\mathbf{E}_1$, $\mathbf{H}_1$ and $\mathbf{E}_2$, $\mathbf{H}_2$ identify two different solutions, we can demonstrate the following result:

Reciprocity theorem

$$\nabla \left(\mathbf{E}_1 \times \mathbf{H}_2^* + \mathbf{E}_2^* \times \mathbf{H}_1\right) = 0, \tag{3}$$

which constitutes the reciprocity theorem of Maxwell equations. The theorem holds for any set of solutons. If we consider $\mathbf{E}_i$, $\mathbf{H}_i$ to be a set of waveguide modes, we then have:

$$\left[\nabla_\perp - i\left(\beta_\nu - \beta_\mu\right)\hat{z}\right]\left(\mathbf{E}_\nu \times \mathbf{H}_\mu^* + \mathbf{E}_\mu^* \times \mathbf{H}_\nu\right) = 0, \tag{4}$$

Mode normalization and orthogonality

Equation (4) has the expression of a conservation law, written in divergence free form. By integrating along the plane (x, y) each member of the equation, we obtain the following condition:

$$i\left(\beta_\nu - \beta_\mu\right) \iint_{-\infty}^{\infty} dxdy \left(\mathbf{P}_{\nu\mu} \cdot \hat{z}\right) = \iint_{-\infty}^{\infty} dxdy \nabla_\perp \mathbf{P}_{\nu\mu} = 0, \qquad (5)$$

where $\mathbf{P}_{\nu\mu} \equiv \mathbf{E}_\nu \times \mathbf{H}_\mu^* + \mathbf{E}_\mu^* \times \mathbf{H}_\nu$ and the last equality $\iint dxdy \nabla_\perp \mathbf{P}_{\nu\mu} = 0$ holds for periodic or constant boundary conditions at infinity $(x, y) \to \infty$. Equation (5) implies the following orthogonality condition among the modes:

Orthonormality condition

$$\frac{1}{4} \iint_{-\infty}^{\infty} dxdy \left(\mathbf{P}_{\nu\mu} \cdot \hat{z}\right) = \delta_{\mu\nu} \qquad (6)$$

which is usually normalized with the factor $\frac{1}{4}$.

Mode normalization and orthogonality

Advanced question: why the factor $\frac{1}{4}$ in the orthogonality condition and what is the physical meaning of Eq. (6)?

Outline

Reference texts

- H. Nishiara, *Optical Integrated Circuits* (McGraw Hill, 1989). Chapters 2 & 3.
- K. Okamoto, *Fundamentals of Optical Waveguides* (Academic Press, 2010). Chapter 1.
- C. A. Balanis, *Advanced Engineering Electromagnetics* (Wiley, 1989). Chapter 7.