EE391D Advanced Topics in Photonics

Waveguide theory
lesson 10

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March 29, 2016

Outline

Multilayer theory

A multilayer waveguide is composed of a series of layers, parallel to the propagation axis z and symmetric along y. Each layer has a thickness h_i and refractive index n_i. By defying the following variables:

$$\begin{cases} U(x) = E_y(x), \\ V(x) = \omega \mu H_z(x), \end{cases} \tag{1}$$

for the transverse profile of TE modes of the structure, Maxwell equations reduce to the following harmonic set:

$$\begin{cases} \frac{dU}{dx} = -iV, \\ \frac{dV}{dx} = i\left(\beta^2 - nk^2\right) U \end{cases} \tag{2}$$

being β the propagation constant of the mode and $k = \frac{2\pi}{\lambda}$ the wavenumber in vacuum.

Multilayer theory

Following the analysis of (Tamir, 2012), the transfer matrix of a stack of N layers is defined as follows:

$$\begin{bmatrix} U_0 \\ V_0 \end{bmatrix} = \mathbf{M} \begin{bmatrix} U_N \\ V_N \end{bmatrix} = \prod_{j=0}^{N} \mathbf{M}_j \begin{bmatrix} U_N \\ V_N \end{bmatrix}, \tag{3}$$

with the characteristic matrix:

$$\mathbf{M}_j = \begin{bmatrix} \cos(k_j h_j) & \frac{i}{k_j}\sin(k_j h_j) \\ ik_j \sin(k_j h_j) & \cos(k_j h_j) \end{bmatrix}, \tag{4}$$

with $k_j = n_j^2 k^2 - \beta^2$. The dispersion relation of guided modes is then:

$$i(\gamma_s m_{11} + \gamma_c m_{22}) - m_{21} + \gamma_s \gamma_c m_{12} = 0, \tag{5}$$

with $\mathbf{M}_{ij} = m_{ij}$, $\gamma_s = \sqrt{\beta^2 - n_s^2 k^2}$, $\gamma_c = \sqrt{\beta^2 - n_c^2 k^2}$ the substrate and cladding decaying constants, respectively, of the mode.

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Example: slab waveguide

In the case of a slab waveguide of index n_g and thickness h, the dispersion relation reads as follows:

$$\kappa h = \tan^{-1}\left(\frac{\frac{\gamma_s}{\kappa}+\frac{\gamma_c}{\kappa}}{1-\frac{\gamma_s\gamma_c}{\kappa^2}}\right)+m\pi, \tag{6}$$

with $\kappa = n_g^2 k^2 - \beta^2$ and $m = 0, \pm 1, \pm 2, ...$ By using trigonometric identities, we can rewrite the dispersion relation in the following form:

$$\kappa h = \tan^{-1}\left(\frac{\gamma_s}{\kappa}\right)+\tan^{-1}\left(\frac{\gamma_c}{\kappa}\right)+m\pi, \tag{7}$$

which furnishes exactly the dispersion relation of TE modes obtained through ray optics analysis in the Optics class (see Lesson 14).

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2d waveguides

From Maxwell equations:

$$\begin{cases} \nabla \times \mathbf{E} = -i\omega\mu\mathbf{H}, \\ \nabla \times \mathbf{H} = i\omega\epsilon\mathbf{E}, \end{cases} \tag{8}$$

we apply a longitudinal-transverse decomposition:

$$\begin{cases} \mathbf{E} = \mathbf{E}_t + \hat{z}E_z, \\ \mathbf{H} = \mathbf{H}_t + \hat{z}H_z, \\ \nabla = \nabla_t + \hat{z}\partial_z, \end{cases} \tag{9}$$

with $\nabla_t = \hat{x}\partial_x + \hat{y}\partial y$ and $\mathbf{E}_t$, $\mathbf{H}_t$ the transverse component of the electric and magnetic field, respectively.

2d waveguides

By looking for modal solutions $\mathbf{E}(x, y, z) = \mathbf{E}(x, y)e^{-i\beta z}$, $\mathbf{H}(x, y, z) = \mathbf{H}(x, y)e^{-i\beta z}$, we obtain:

$$\begin{cases} \nabla_t E_z \times \hat{z} - i\beta\hat{z} \times \mathbf{E}_t = -i\omega\mu\mathbf{H}_t, \\ \nabla_t H_z \times \hat{z} - i\beta\hat{z} \times \mathbf{H}_t = i\omega\epsilon\mathbf{E}_t, \\ \nabla_t \times \mathbf{E}_t + i\omega\mu\hat{z}H_z = 0, \\ \nabla_t \times \mathbf{H}_t - i\omega\epsilon\hat{z}E_z = 0, \end{cases} \tag{10}$$

The solution of these set of equations allow to calculate the modal eigenvalue β and the eigenvectors **E** and **H**. This is typically refereed as the full vectorial mode solution of the problem.

Outline

quasi-TE and quasi-TM modes

In dielectric waveguides, unlike classical conductor waveguides, there are in general no simple guided mode solutions (TEM, TE or TM) with some components of the field equal to 0. However, high refractive index structure typically possess mode with a dominant transverse component:

$$\begin{cases} E_z \lll |\mathbf{E}_t|, \\ H_z \lll |\mathbf{H}_t|, \end{cases} \tag{11}$$

thus generating modes with a transverse electric and magnetic field profile and negligible component along the propagation direction. These modes are known as quasi-TE and quasi-TM, generalizing TE and TM modes of planar structures. By neglecting longitudinal components E_z and H_z, the modal equations (10) simplify in a semi-vectorial set. This set is different for x-polarized and y-polarized solutions, where the electric field is discontinuous at the boundaries between different media along x and y, respectively. These discontinuities are in general small, and if they are neglected the modal equations simplify even further into a single scalar equation.

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Exercises in 1D and 2D waveguide structures

Please refer to the matlab codes @multi for 1D structures and @waveguide for 2D waveguides in scalar, semi-vectorial and full-vectorial analysis downloadable from the website www.primalight.org.

Outline

Reference texts

- H. Nishiara, *Optical Integrated Circuits* (McGraw Hill, 1989). Chapters 2 & 3.
- K. Okamoto, *Fundamentals of Optical Waveguides* (Academic Press, 2010). Chapter 1.
- C. A. Balanis, *Advanced Engineering Electromagnetics* (Wiley, 1989). Chapter 7.
- T. Tamir, *Guided-wave optoelectronics* (Springer, 2012).