

EE391D Advanced Topics in Photonics

Light propagation in anisotropic crystals lesson 6

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Outline

- 1 The Electro-Optic modulation of light
 - The Electro-Optic tensor
- 2 Electro-Optic modulators with KDP crystals
- 3 Reference texts

The Electro-Optic tensor

In the principal axis representation, the index ellipsoid for an anisotropic crystal is as follows:

$$\frac{x^2}{n_1^2} + \frac{y^2}{n_2^2} + \frac{z^2}{n_3^2} = 1. \quad (1)$$

For an electro-optic material, the application of an electric field \mathbf{E}_0 modifies the dielectric tensor and the index ellipsoid:

$$A_1x^2 + A_2y^2 + A_3z^2 + 2A_4yz + 2A_5xz + 2A_6xy = 1, \quad (2)$$

with coefficients $A_i = A_{i0} + \Delta A_i = A_{i0} + r_{ij}E_{0j} = \underline{\underline{r}} \cdot \mathbf{E}_0$. The 6×3 tensor $\underline{\underline{r}}$ is called **electro-optic tensor** and its form depends on the symmetry class of the material. The coefficients r_{ij} of the tensor are available in the literature for different electro-optic media.

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Electro-Optic modulators with KDP crystals

The KDP is a uniaxial crystal that is well used to develop photonics applications with electro-optic materials. KDP belongs to the symmetry group $\bar{4}2m$. The only nonzero elements of the electro-optics tensor of KDP are $r_{52} = r_{41}$ and r_{63} . After the application of a static field \mathbf{E}_0 to a KDP crystal with optical axis along z , the index ellipsoid becomes:

$$\frac{x^2}{n_o^2} + \frac{y^2}{n_o^2} + \frac{z^2}{n_e^2} + 2r_{41}E_{0x}yz + 2r_{41}E_{0y}xz + 2r_{63}E_{0z}xy = 1 \quad (3)$$

If the static field $\mathbf{E}_0 = E_z \hat{z}$ is applied parallel to z , we have:

$$\frac{x^2}{n_o^2} + \frac{y^2}{n_o^2} + \frac{z^2}{n_e^2} + 2r_{63}E_{0z}xy = 1 \quad (4)$$

Question: how the ellipsoid have changed?

Electro-Optic modulators with KDP crystals

We can write (4) as the following quadratic form:

$$\begin{bmatrix} x & y & z \end{bmatrix} \begin{bmatrix} \frac{1}{n_o^2} & r_{63}E_{0z} & 0 \\ r_{63}E_{0z} & \frac{1}{n_o^2} & 0 \\ 0 & 0 & \frac{1}{n_e^2} \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \mathbf{r}^\dagger \underline{\underline{A}} \mathbf{r} = 1, \quad (5)$$

which can be easily diagonalized from the eigenvalues and eigenvectors of the symmetric matrix $\underline{\underline{A}}$:

$$\underline{\underline{A}}\phi_j = \lambda_j\phi_j, \quad \begin{cases} \lambda_1 = \frac{1}{n_o^2} + r_{63}E_{0z}, \phi_1 = \left[\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0\right] \\ \lambda_2 = \frac{1}{n_o^2} - r_{63}E_{0z}, \phi_2 = \left[-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0\right] \\ \lambda_3 = \frac{1}{n_e^2}, \phi_3 = [0, 0, 1], \end{cases} \quad (6)$$

by using the following coordinate transform $\mathbf{r} = \underline{\underline{T}}\mathbf{r}_1$, with $\underline{\underline{T}} = [\phi_1^\dagger, \phi_2^\dagger, \phi_3^\dagger]$.

Electro-Optic modulators with KDP crystals

By direct substitution:

$$\underline{\underline{\mathbf{r}^\dagger \mathbf{A} \mathbf{r}}} = \underline{\underline{\mathbf{r}_1^\dagger \mathbf{T}^\dagger \mathbf{A} \mathbf{T} \mathbf{r}_1}} = \underline{\underline{\mathbf{r}_1^\dagger}} \begin{bmatrix} \frac{1}{n_o^2} + r_{63} E_{0z} & 0 & 0 \\ 0 & \frac{1}{n_o^2} - r_{63} E_{0z} & 0 \\ 0 & 0 & \frac{1}{n_e^2} \end{bmatrix} \underline{\underline{\mathbf{r}_1}} = 1, \quad (7)$$

we obtain the index ellipsoid in the new principal axis representation:

$$\left(\frac{1}{n_o^2} + r_{63} E_{0z} \right) x_1^2 + \left(\frac{1}{n_o^2} - r_{63} E_{0z} \right) y_1^2 + \frac{z^2}{n_e^2} = 1. \quad (8)$$

The coordinate transformation:

$$\underline{\underline{\mathbf{T}}} = \begin{bmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \cos \theta & -\sin \theta & 0 \\ \sin \theta & \cos \theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad (9)$$

is a counterclockwise rotation of $\theta = 45$ degrees in (x, y) .

Electro-Optic modulators with KDP crystals

The application of the electro-optic effect turns the uniaxial KDP into a biaxial crystal, as seen from (8). The refractive indices along the new principal axis (x_1, y_1) read:

$$\begin{aligned}\frac{1}{n_{x1}^2} &= \frac{1}{n_o^2} + r_{63}E_{0z}, \\ \frac{1}{n_{y1}^2} &= \frac{1}{n_o^2} - r_{63}E_{0z},\end{aligned}\tag{10}$$

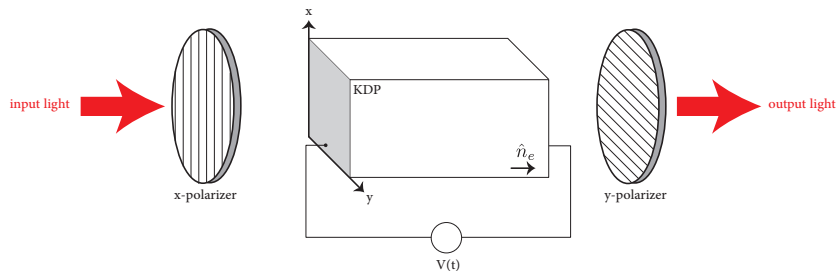
which leads to $(r_{63}E_{0z} \lll 1)$:

$$\begin{aligned}n_{x1} &= n_o - \frac{n_o^3}{2}r_{63}E_{0z}, \\ n_{y1} &= n_o + \frac{n_o^3}{2}r_{63}E_{0z}.\end{aligned}\tag{11}$$

These expressions are the electro-optic refractive index shifts due to the application of the field E_{0z} to the KDP crystal.

Electro-Optic modulators with KDP crystals

The following system describes the basic prototype of light modulator system that can be designed with a KDP crystal.



It is composed by two polarizers in cross configuration, and a KDP crystal with optical axis along z (z -cut) and voltage applied on the input/output facets of the crystal.

Electro-Optic modulators with KDP crystals

We can analyze the system by using the Jones matrices. We have the following elements:

$$\begin{bmatrix} E_x(L) \\ E_y(L) \end{bmatrix} = \underline{\underline{P}}_y \underline{\underline{R}}^\dagger e^{-ikLn} \underline{\underline{R}} \underline{\underline{P}}_x \begin{bmatrix} E_x(0) \\ E_y(0) \end{bmatrix}, \quad n = \begin{bmatrix} n_{x1} & 0 \\ 0 & n_{y1} \end{bmatrix} \quad (12)$$

with $\underline{\underline{R}}$ a rotation of coordinates of $\frac{\pi}{4}$, and $\underline{\underline{P}}_i$ a polarizer along the direction i . By calculating the various product, we obtain the transfer function of the system:

$$\begin{bmatrix} E_x(L) \\ E_y(L) \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ \frac{1}{2} (e^{-ikn_{y1}L} - e^{-ikn_{x1}L}) & 0 \end{bmatrix} \begin{bmatrix} E_x(0) \\ E_y(0) \end{bmatrix}, \quad (13)$$

which leads to $E_y(L) = \frac{1}{2} e^{-ikn_{y1}L} (1 - e^{ik\Delta nL}) E_x(0)$, with $\Delta n = n_{y1} - n_{x1}$.

Electro-Optic modulators with KDP crystals

The output intensity $I_o = |E_y|^2$ reads as follows:

$$I_o = \frac{1}{4} (2 - 2 \cos k\Delta nL) = \sin^2 \left(\frac{k\Delta nL}{2} \right) = \sin^2 \left(\frac{\pi}{2} \frac{V}{V_\pi} \right), \quad (14)$$

with:

$$\begin{cases} V = E_{0z}L, \\ V_\pi = \frac{\lambda}{2n_o^3 r_{63}}. \end{cases} \quad (15)$$

By using a time modulation on the applied voltage $V(t) = V_0 + V_m(t)$ around a working point V_0 , we can modulate the output intensity of the optical signal. This device acts an **electro-optic amplitude modulator**. The main advantage of this optical modulation consists in the fact that the electro-optic response of many semiconductor materials is on the optical fs scale, allowing to achieve ultrafast signal modulations.

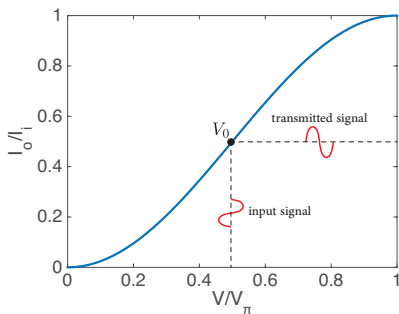
Electro-Optic modulators with KDP crystals

Question: What value of V_0 would you choose for an optimal amplitude modulation?

Electro-Optic modulators with KDP crystals

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The main issue for an amplitude modulator is to provide the minimal distortion to the input signal. To this extent, we can use $V_0 = \frac{V_\pi}{2}$, which provides a working region in the input-output curve of the device where the transfer function $\sin^2(\pi V/2V_\pi)$ is well approximated by a linear slope for small input variations of $V_m(t)$.



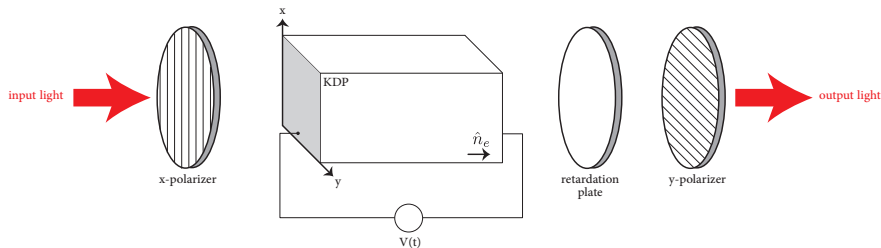
Electro-Optic modulators with KDP crystals

Question: How to physically implement the condition $V_0 = \frac{V_\pi}{2}$?

Electro-Optic modulators with KDP crystals

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For $V_0 = \frac{V_\pi}{2}$, the phase difference $k\Delta nL = \frac{\pi}{2}$. This can be easily implemented by adding a quarter-wave retardation plate at the crystal output.



Exercise: calculate the length D along z of the retarder plate to achieve the required condition on V_0 at the output.

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Reference texts

- A. Yariv, *Photonics: Optical Electronics in Modern Communications* (Oxford University Press, 2006). Chapter 9
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- R. W. Boyd, *Nonlinear Optics* (Academic Press, 2008). Chapter 1 and 11