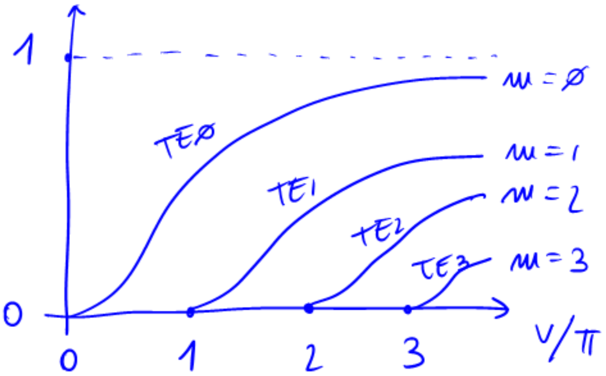
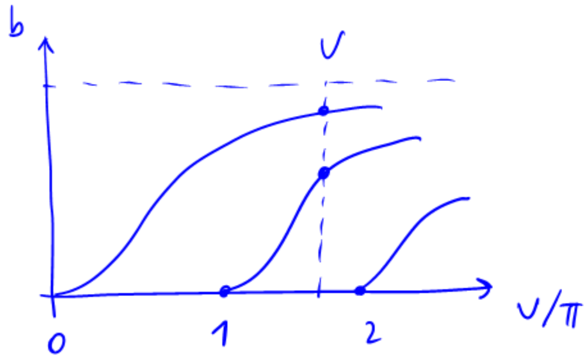
Lesson 14 ⟷ 15 - NOV - 2011

Exercises on symmetric waveguides:

1) $b(V)$: $V\sqrt{1-b} = m\pi + 2\,tg^{-1}\sqrt{\frac{b}{1-b}}$



2) Number of modes for a given V.



since $V(0) = m\pi$, the number of modes is

$N = INT\left(\frac{V}{\pi}\right)$

A very important physical observation is as follows. while high order modes exist above a cutoff frequency, the fundamental mode TE∅ is always present regardless of the geometry of the problem.

3) TM Modes. By applying the same algebra, we end with:

$$V\sqrt{1-b} = m\pi + 2\,tg^{-1}\left[\frac{n_p^2}{n_1^2}\sqrt{\frac{b}{1-b}}\right]$$

which is very close to the TE case, but is no longer universal as it contains the parameter $\frac{n_f^2}{n_1^2}$. However, in many cases $n_f \cong n_1$, hence, at first approximation the TE curve can be used.

ASYMMETRIC WAVEGUIDE: TE CASE

by introducing $\xi = \cos\theta_i$, we have:

$$2 n_f \xi k h = m\pi + tg^{-1}\sqrt{\frac{n_f^2 - n_1^2 - n_f^2\xi^2}{n_f^2\xi^2}} + tg^{-1}\sqrt{\frac{n_f^2 - n_1^2 + n_1^2 - n_2^2 - n_2^2\xi^2}{n_f^2\xi^2}}$$

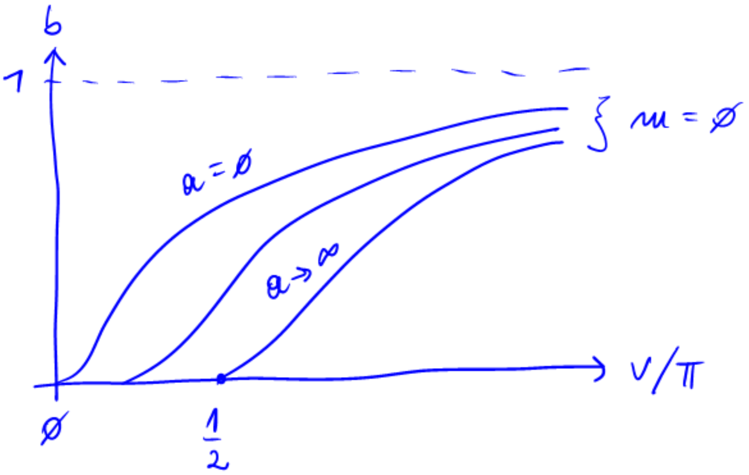
we can express both sides of the equation as a function of $\alpha = n_f^2\xi^2/(n_f^2 - n_1^2)$, as done before:

$$V\sqrt{\alpha} = m\pi + tg^{-1}\sqrt{\frac{1-\alpha}{\alpha}} + tg^{-1}\sqrt{\frac{1-\alpha+a}{\alpha}}$$

with $a = \frac{n_1^2 - n_2^2}{n_f^2 - n_1^2}$ playing the role of an ASYMMETRY PARAMETER

by introducing the NORMALIZED INDEX $b = 1-\alpha$, we have:

$$V\sqrt{1-b} = m\pi + tg^{-1}\sqrt{\frac{b}{1-b}} + tg^{-1}\sqrt{\frac{a+b}{1-b}} \quad , \quad m = 0, 1, \dots$$



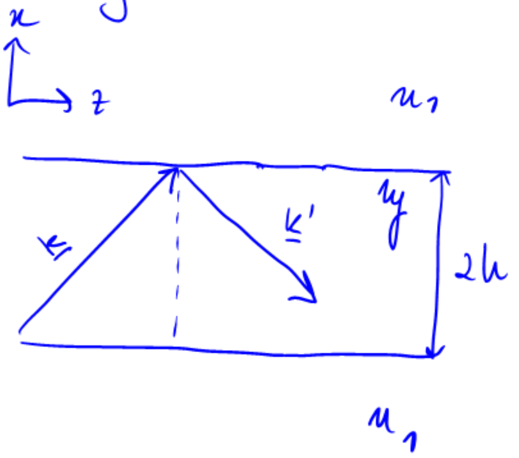
$$V(0) = m\pi + tg^{-1}\sqrt{a}$$

$$\lim_{a\to\infty} V(0) = \pi(m + 1/2)$$

if $a > 0$, a cutoff frequency exists also for TE_0

Modal profile

The last question we need to address is the profile of the guided modes. We will begin by the TE case:



inside the slab, the field is composed by two plane waves with the same k, but with opposite orientations of $\underline{k}$ along x:

INSIDE → $\underline{E} = E\cdot\hat{y}$; $E = \frac{a}{2}\cdot e^{jk_x\cdot x} + \frac{b}{2}e^{-jk_x x} = A\cos(k_x x+\varphi)$

$n_0^2 k^2 = k_x^2 + k_z^2 = k_x^2 + \beta^2$; $k_x = \sqrt{n_0^2k^2-\beta^2} = \gamma$

therefore:

INSIDE → $E = A\cos(\gamma x+\varphi)$; for simmetry reasons: $\varphi = 0, \frac{\pi}{2} \Rightarrow E = \begin{cases} A\cos\gamma x \\ A\sin\gamma x \end{cases}$

Outside the waveguide, TIR imposes the presence of evanescent fields of the type:

$Be^{\pm jk'_x\cdot x}$, with $k'_x = \sqrt{n_1k^2-\beta^2}$, but $\beta^2 > n_1^2k^2$

cause $\beta = n_{eff}\cdot k$, hence, $\beta^2 = n_{eff}^2\cancel{k^2} > n_1^2\cancel{k^2}$

and therefore:

$k'_x = \pm\sqrt{\beta^2 - n_1^2k^2} = \pm\delta$. the field is then:

$E = B\cdot e^{-\delta(|x|-h)}$

SUMMARIZING:

TE EVEN

$$E = \begin{cases} A \cos \gamma x , & |x| < h \\ B e^{-\delta(|x|-h)} , & |x| > h \end{cases}$$

TE ODD

$$E = \begin{cases} A \sin \gamma x , & |x| < h \\ B' e^{-\delta(|x|-h)} , & |x| > h \end{cases}$$

$\hookrightarrow B \cdot \text{sign}(x)$

- Question: How to determine the value of the constants A and B?

We need to impose the continuity of the field

MODE TE EVEN

$$A \cos \gamma h = B$$

MODE TE ODD

$$A \sin \gamma h = B$$

We therefore have:

$$V \cdot \sqrt{1-b} = m \cdot \pi + 2 \, tg^{-1} \sqrt{\frac{b}{1-b}}$$

$m = 2n$ TE EVEN

$m = 2n+1$ TE ODD

$n = 0, 1, \ldots$

$$E = \begin{cases} B \dfrac{\cos \gamma x}{\cos \gamma h} , & |x| < h \\ B \cdot e^{-\delta(|x|-h)} , & |x| > h \end{cases} ; \quad E = \begin{cases} B \cdot \dfrac{\sin \gamma x}{\sin \gamma h} , & |x| < h \\ B \cdot \text{sign}(x) \cdot e^{-\delta(|x|-h)} , & |x| > h \end{cases}$$

TE EVEN

$$\gamma = \sqrt{n_g^2 k^2 - \beta^2} = \frac{2\pi}{\lambda} \sqrt{n_g^2 - n_{eff}^2}$$

TE ODD

$$\delta = \frac{2\pi}{\lambda} \sqrt{n_{eff}^2 - n_1^2}$$

- EXERCISE: write a Matlab program that, for given λ, n_1, n_g, h, calculate the modes indices n_{eff} and plot the mode profiles.