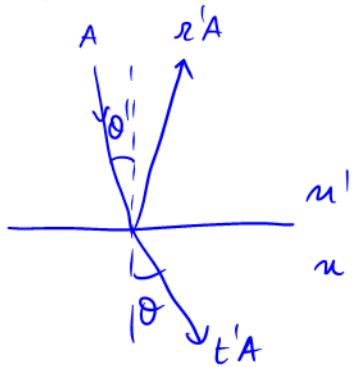
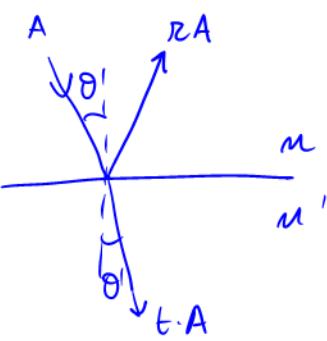
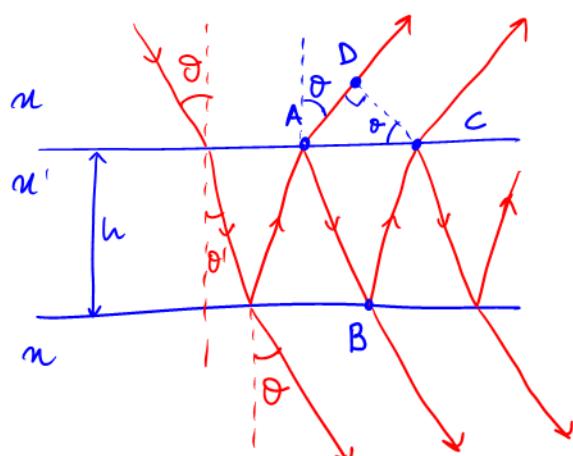


Fabry-Pérot resonator



Phase shift between 2 consecutive rays is $\delta = k_0 \cdot \Delta s$

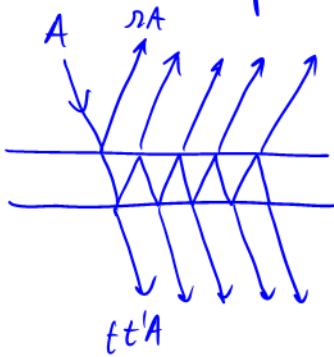
$$\begin{aligned}\Delta s &= \text{optical path difference} = \\ &= n'(AB \cdot 2) - nAD = \frac{n \cdot 2h}{\cos \theta}, -nAC \cdot \sin \theta = \\ &= \frac{n \cdot 2h}{\cos \theta} - 2h \cdot \tan \theta \cdot n \cdot \sin \theta = \frac{n \cdot 2h}{\cos \theta}, -2h n' \cdot \sin \theta / \tan \theta \\ &= 2n'h \left(\frac{1 - \sin^2 \theta'}{\cos \theta'} \right) = 2n'h \cdot \cos \theta'\end{aligned}$$

$$\text{or equivalently: } \delta = k \cdot \hat{n} \cdot 2h = 2n' k \cos \theta' h$$

r, t are reflection and transmission coefficients for a plane wave propagating from

a dielectric with index n to a material with n' . r' and t' are the same set of coefficients for the opposite situation.

We therefore have:



Reflections:	rA	first ray
	$tt'r'Ae^{i\delta}$	second ray
	$tt'r'^2Ae^{i\delta}$	third ray
	$tt'r'^{(p-1)}Ae^{i(p-1)\delta}$	p-th ray

Transmissions:

$tt'A$	first ray
$tt'r'^2Ae^{i\delta}$	second ray
$tt'r'^{(p-1)}A \cdot e^{i(p-1)\delta}$	p-th ray

(2)

r, r', t, t' are expressed by the FRESNEL FORMULAE.
As can be verified by straightforward substitution,
we have:

$$tt' = T \quad \text{TRANSMISSIVITY}$$

$$r^2 = r'^2 = R \quad \text{REFLECTIVITY}, \text{ & } r = -r'$$

$$R + T = 1 \quad \text{conservation of Energy}$$

Let us now calculate the reflected A_r and transmitted
At amplitudes:

$$A_r = [r + tt'r'e^{i\delta} (1 + r'^2 e^{i\delta} + \dots + r'^2 (p-2) e^{i(p-2)\delta})] A =$$

$$= [r + tt'r'e^{i\delta} \sum_{p=2}^{\infty} r'^2 (p-2) \cdot e^{i(p-2)\delta}] A =$$

$$= [r + tt'r'e^{i\delta} \cdot \frac{1}{1 - r'^2 e^{i\delta}}] A =$$

$$= [r + \frac{T \cdot r' e^{i\delta}}{1 - R e^{i\delta}}] A = \frac{r(1 - R e^{i\delta}) + T r' e^{i\delta}}{1 - R e^{i\delta}} A =$$

$$= r' \frac{-1 + R e^{i\delta} + T e^{i\delta}}{1 - R e^{i\delta}} = -r' \frac{1 - e^{i\delta}}{1 - R e^{i\delta}} \cdot A$$

The Intensity of the reflected light:

$$I = |A_r|^2 = \frac{(2 - 2 \cos \delta) R}{1 + R^2 - 2 R \cos \delta} I_i = \frac{4 R \sin^2 \delta / 2}{(1 - R)^2 + 4 R \sin^2 \delta / 2} I_i$$

with $I_i = |A|_i^2$

③

for the transmitted Intensity, we can repeat the same analysis done before, or use the conservation of energy:

$$I_t = |A_t|^2 = I_i - I_R = I_i \left(1 - \frac{4R \sin^2 \delta/2}{T^2 + 4R \sin^2 \delta/2} \right) = \\ = I_i \frac{T^2}{(1-R^2) + 4R \sin^2 \delta/2}$$

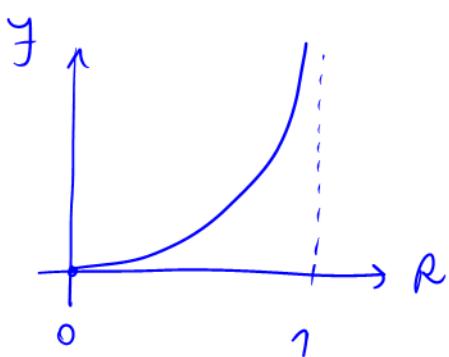
we therefore have:

$$\frac{I_R}{I_i} = \frac{4R \sin^2 \delta/2}{T^2 + 4R \sin^2 \delta/2} \quad \text{ff} \quad \frac{I_t}{I_i} = \frac{T^2}{T^2 + 4R \sin^2 \delta/2}$$

which are known as Airy's formulae. These can be written by employing a more useful parameter $F = \frac{4R}{(1-R)^2}$

$$\frac{I_R}{I_i} = \frac{F \sin^2 \delta/2}{1 + F \sin^2 \delta/2} \quad ; \quad \frac{I_t}{I_i} = \frac{1}{1 + F \sin^2 \delta/2}$$

let us now study the behavior of I_t/I_i :



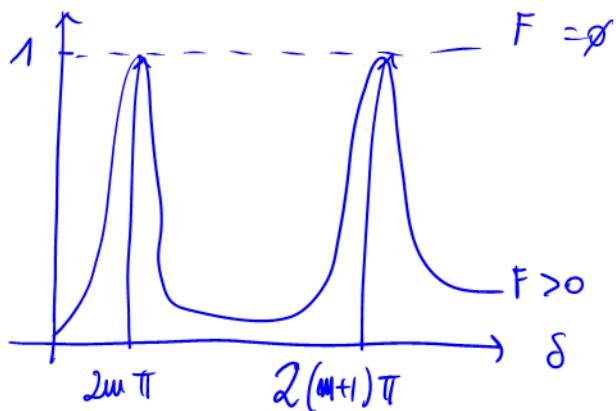
the F function maps R on a larger domain, so is much more sensitive to small variations of R .

$$\text{for } F \rightarrow \phi \Rightarrow I_t/I_i = 1$$

$$\text{for } F \rightarrow \infty \quad F \sin^2 \delta/2 = \begin{cases} \infty & \text{for } \delta/2 \neq m\pi \\ \phi & \text{for } \delta/2 = m\pi \end{cases}$$

we therefore have:

(4)



the FWHM of the fringes is $\delta^* = 2m\pi \pm \varepsilon/2$

$$\frac{1}{1 + F \sin^2 \varepsilon / L} = \frac{1}{2}$$

$$\text{for } F \gg 1 \quad \sin^2 \varepsilon / 2 \cong (\varepsilon / 2)^2$$

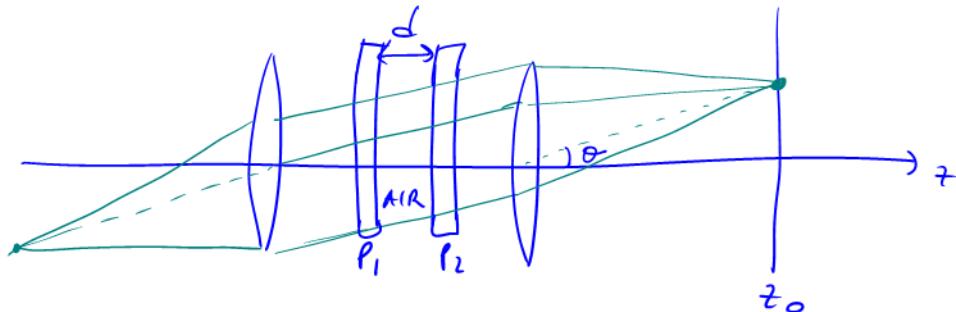
and we get:

$$\varepsilon = \frac{4}{\sqrt{F}}$$

the ratio between the fringe separation and the FWHM is called FINESSE γ

$$\gamma = \frac{2\pi}{\varepsilon} = \frac{\pi \sqrt{F}}{2} \Rightarrow F = \left(\frac{2\gamma}{\pi}\right)^2$$

APPLICATION : FABRY-PEROT INTERFEROMETER



P_1, P_2 = quartz or glass plates of high reflectivity

the condition of maxima in the transmission is:

$$\delta = 2m\pi = 2d \cos \theta \cdot k = \frac{4\pi d \cos \theta}{\lambda}$$

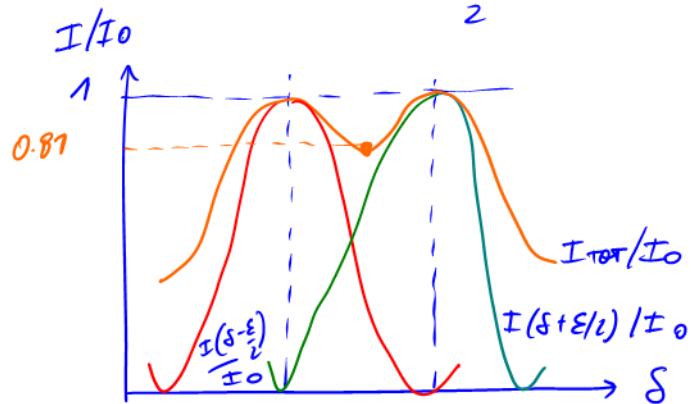
depends on the wavelength λ - let us calculate the resolving power of the system:

let us consider two wavelengths λ_1, λ_2 that leads to $\delta + \frac{\varepsilon}{2}$, $\delta - \frac{\varepsilon}{2}$ being $f(\Delta\theta)$ with $\Delta\theta = \frac{\lambda_1 + \lambda_2}{2}$

the total intensity will be :

$$I_{\text{tot}}(\delta, \varepsilon) = I(\delta + \frac{\varepsilon}{2}) + I(\delta - \frac{\varepsilon}{2}) =$$

$$= \frac{I_0}{1 + F \sin^2 \frac{\delta + \frac{\varepsilon}{2}}{2}} + \frac{I_0}{1 + F \sin^2 \frac{\delta - \frac{\varepsilon}{2}}{2}}$$



RAYLEIGH CRITERION \rightarrow maximum fringe separation detectable is when one maximum coincides with the first zero of the other, or equivalently, when the combined intensity at the mid point is $\frac{8}{\pi^2} = 0.81$

in our case :

$$I_{\text{tot}}(0, \varepsilon) = \frac{2 I_0}{1 + F \sin^2 \frac{\varepsilon}{2}} = 0.81 \cdot \left[I_0 + \frac{I_0}{1 + F \sin^2 \frac{\varepsilon}{2}} \right] = 0.81 \cdot I_{\text{tot}}\left(\frac{\varepsilon}{2}, \varepsilon\right)$$

$$\text{For } F \gg 1, \quad \varepsilon \approx \frac{2\pi}{f}$$

$$\text{now, } \delta = \frac{4\pi h \cos \theta}{\lambda_0} ; \quad \Delta\delta = \frac{4\pi h \cos \theta}{\lambda_0^2}. \Delta\theta = 2\pi m \cdot \frac{\Delta\lambda}{\lambda_0}$$

$$\frac{\lambda_0}{\Delta\lambda} = \frac{2\pi m}{\Delta\delta} = \frac{2\pi m}{\varepsilon} \approx \frac{2\pi m}{2\pi} f = m f$$

(6)

for normal incidence :

$$\cos \theta \approx 1 \Rightarrow \delta = \frac{4\pi}{\lambda_0} \cdot h = 2m\pi \Rightarrow m = \frac{2h}{\lambda_0}$$

and therefore we get :

$$\frac{\lambda_0}{\Delta\lambda} = \frac{2h\gamma}{\lambda_0}$$

example : $R = 0.9 \rightarrow \gamma = 30$

$$h = 1 \text{ cm}$$

$$\lambda_0 = 0.5 \mu\text{m}$$

we have :

$$\frac{\lambda_0}{\Delta\lambda} = \frac{2 \cdot 30 \cdot 10^{-2}}{0.5 \cdot 10^{-6}} = 60 \cdot 2 \cdot 10^9 = 1.2 \cdot 10^6$$

$$\frac{\Delta\lambda}{\lambda_0} = 0.6 \cdot 10^{-6} \quad \text{extremely high}$$

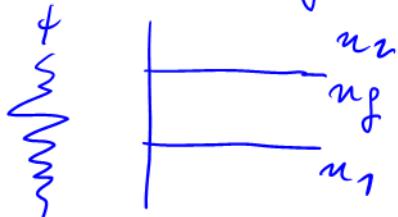
the wavelength difference $(\Delta\lambda)_s$ corresponding to a displacement of one order $|\Delta\delta| = 2\pi$ is called the FREE-SPECTRAL RANGE ; near normal incidence is :

$$(\Delta\lambda)_s = \frac{\lambda_0}{m} \approx \frac{\lambda_0^2}{2h}$$

for the same values as before, we get :

$$\frac{(0.5 \cdot 10^{-6})^2}{2 \cdot 10^{-2}} = 0.012 \text{ mm}, \text{ very small!}$$

) Exercise : given a slab waveguide and an input field ψ



how to calculate how much energy is coupled to guided modes ?