ECE325 Advanced Photonics

Light propagation in anisotropic crystals
lesson 6

Andrea Fratalocchi

www.primalight.org

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Outline

1. The Electro-Optic modulation of light
   - The Electro-Optic tensor
2. Electro-Optic modulators with KDP crystals
3. Reference texts

The Electro-Optic tensor

In the principal axis representation, the index ellipsoid for an anisotropic crystal is as follows:

$$\frac{x^2}{n_1^2} + \frac{y^2}{n_2^2} + \frac{z^2}{n_3^2} = 1. \quad (1)$$

For an electro-optic material, the application of an electric field $\boldsymbol{E}_0$ modifies the dielectric tensor and the index ellipsoid:

$$A_1 x^2 + A_2 y^2 + A_3 z^2 + 2A_4 yz + 2A_5 xz + 2A_6 xy = 1, \quad (2)$$

with coefficients $A_i = A_{i0} + \Delta A_i = A_{i0} + r_{ij} E_{0j} = \underline{\underline{r}} \cdot \boldsymbol{E}_0$. The 6×3 tensor $\underline{\underline{r}}$ is called electro-optic tensor and its form depends on the symmetry class of the material. The coefficients r_{ij} of the tensor are available in the literature for different electro-optic media.

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Electro-Optic modulators with KDP crystals

The KDP is a uniaxial crystal that is well used to develop photonics applications with electro-optic materials. KDP belongs to the symmetry group $\bar{4}2m$. The only nonzero elements of the electro-optics tensor of KDP are $r_{52} = r_{41}$ and r_{63}. After the application of a static field $\boldsymbol{E}_0$ to a KDP crystal with optical axis along z, the index ellipsoid becomes:

$$\frac{x^2}{n_o^2} + \frac{y^2}{n_o^2} + \frac{z^2}{n_e^2} + 2r_{41}E_{0x}yz + 2r_{41}E_{0y}xz + 2r_{63}E_{0z}xy = 1 \tag{3}$$

If the static field $\boldsymbol{E}_0 = E_z\hat{z}$ is applied parallel to z, we have:

$$\frac{x^2}{n_o^2} + \frac{y^2}{n_o^2} + \frac{z^2}{n_e^2} + 2r_{63}E_{0z}xy = 1 \tag{4}$$

Question: how the ellipsoid have changed?

Electro-Optic modulators with KDP crystals

We can write (4) as the following quadratic form:

$$\begin{bmatrix} x & y & z \end{bmatrix} \begin{bmatrix} \frac{1}{n_o^2} & r_{63}E_{0z} & 0 \\ r_{63}E_{0z} & \frac{1}{n_o^2} & 0 \\ 0 & 0 & \frac{1}{n_e^2} \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \boldsymbol{r}^\dagger \underline{\underline{A}} \boldsymbol{r} = 1, \qquad (5)$$

which can be easily diagonalized from the eigenvalues and eigevectors of the symmetric matrix $\underline{\underline{A}}$:

$$\underline{\underline{A}}\phi_j = \lambda_j \phi_j, \qquad \begin{cases} \lambda_1 = \frac{1}{n_o^2} + r_{63}E_{0z}, \phi_1 = [\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0] \\ \lambda_2 = \frac{1}{n_o^2} - r_{63}E_{0z}, \phi_2 = [-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0] \\ \lambda_3 = \frac{1}{n_e^2}, \phi_3 = [0, 0, 1], \end{cases} \qquad (6)$$

by using the following coordinate transform $\boldsymbol{r} = \underline{\underline{T}}\boldsymbol{r}_1$, with $\underline{\underline{T}} = [\phi_1^\dagger, \phi_2^\dagger, \phi_3^\dagger]$.

Electro-Optic modulators with KDP crystals

By direct substitution:

$$\boldsymbol{r}^\dagger \underline{\underline{A}} \boldsymbol{r} = \boldsymbol{r}_1^\dagger \underline{\underline{T}}^\dagger \underline{\underline{A}}\,\underline{\underline{T}} \boldsymbol{r}_1 = \boldsymbol{r}_1^\dagger \begin{bmatrix} \frac{1}{n_o^2} + r_{63}E_{0z} & 0 & 0 \\ 0 & \frac{1}{n_o^2} - r_{63}E_{0z} & 0 \\ 0 & 0 & \frac{1}{n_e^2} \end{bmatrix} \boldsymbol{r}_1 = 1, \tag{7}$$

we obtain the index ellipsoid in the new principal axis representation:

$$\left(\frac{1}{n_o^2} + r_{63}E_{0z}\right) x_1^2 + \left(\frac{1}{n_o^2} - r_{63}E_{0z}\right) y_1^2 + \frac{z^2}{n_e^2} = 1. \tag{8}$$

The coordinate transformation:

$$\underline{\underline{T}} = \begin{bmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{9}$$

is a counterclockwise rotation of $\theta = 45$ degrees in (x, y).

Electro-Optic modulators with KDP crystals

The application of the electro-optic effect turns the uniaxial KDP into a biaxial crystal, as seen from (8). The refractive indices along the new principal axis (x_1, y_1) read:

$$\frac{1}{n_{x1}^2} = \frac{1}{n_o^2} + r_{63}E_{0z},$$
$$\frac{1}{n_{y1}^2} = \frac{1}{n_o^2} - r_{63}E_{0z}, \tag{10}$$

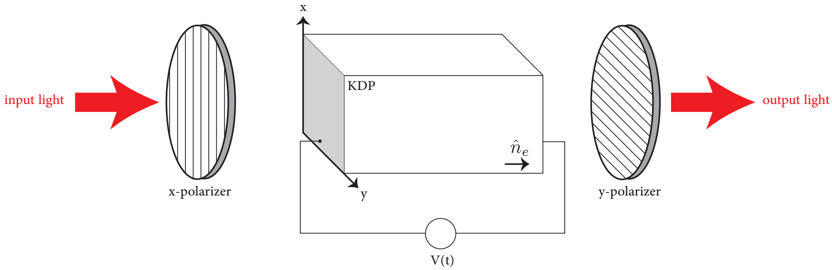
which leads to $(r_{63}E_{0z} \lll 1)$:

$$n_{x1} = n_o - \frac{n_o^3}{2}r_{63}E_{0z},$$
$$n_{y1} = n_o + \frac{n_o^3}{2}r_{63}E_{0z}. \tag{11}$$

These expressions are the electro-optic refractive index shifts due to the application of the field E_{0z} to the KDP crystal.

Electro-Optic modulators with KDP crystals

The following system describes the basic prototype of light modulator system that can be designed with a KDP crystal.



It is composed by two polarizers in cross configuration, and a KDP crystal with optical axis along z (z-cut) and voltage applied on the input/output facets of the crystal.

Electro-Optic modulators with KDP crystals

We can analyze the system by using the Jones matrices. We have the following elements:

$$\begin{bmatrix} E_x(L) \\ E_y(L) \end{bmatrix} = \underline{\underline{P}}_y \underline{\underline{R}}^\dagger e^{-ikL\underline{\underline{n}}} \underline{\underline{R}}\,\underline{\underline{P}}_x \begin{bmatrix} E_x(0) \\ E_y(0) \end{bmatrix}, \qquad n = \begin{bmatrix} n_{x1} & 0 \\ 0 & n_{y1} \end{bmatrix} \tag{12}$$

with $\underline{\underline{R}}$ a rotation of coordinates of $\frac{\pi}{4}$, and $\underline{\underline{P}}_i$ a polarizer along the direction i. By calculating the various product, we obtain the transfer function of the system:

$$\begin{bmatrix} E_x(L) \\ E_y(L) \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ \frac{1}{2}\left(e^{-ikn_{y1}L} - e^{-ikn_{x1}L}\right) & 0 \end{bmatrix} \begin{bmatrix} E_x(0) \\ E_y(0) \end{bmatrix}, \tag{13}$$

which leads to $E_y(L) = \frac{1}{2} e^{-ikn_{y1}L}\left(1 - e^{ik\Delta nL}\right) E_x(0)$, with $\Delta n = n_{y1} - n_{x1}$.

Electro-Optic modulators with KDP crystals

The output intensity $I_o = |E_y|^2$ reads as follows:

$$I_o = \frac{1}{4}(2 - 2\cos k\Delta nL) = \sin^2\left(\frac{k\Delta nL}{2}\right) = \sin^2\left(\frac{\pi}{2}\frac{V}{V_\pi}\right), \qquad (14)$$

with:

$$\begin{cases} V = E_{0z}L, \\ V_\pi = \frac{\lambda}{2n_o^3 r_{63}}. \end{cases} \qquad (15)$$

By using a time modulation on the applied voltage $V(t) = V_0 + V_m(t)$ around a working point V_0, we can modulate the output intensity of the optical signal. This device acts an electro-optic amplitude modulator. The main advantage of this optical modulation consists in the fact that the electro-optic response of many semiconductor materials is on the optical *fs* scale, allowing to achieve ultrafast signal modulations.

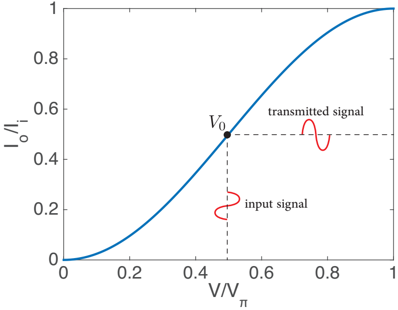
Electro-Optic modulators with KDP crystals

Question: What value of V_0 would you choose for an optimal amplitude modulation?

Electro-Optic modulators with KDP crystals

Question: What value of V_0 would you choose for an optimal amplitude modulation?

The main issue for an amplitude modulator is to provide the minimal distortion to the input signal. To this extent, we can use $V_0 = \frac{V_\pi}{2}$, which provides a working region in the input-output curve of the device where the transfer function $\sin^2(\pi V/2V_\pi)$ is well approximated by a linear slope for small input variations of $V_m(t)$.



Electro-Optic modulators with KDP crystals

Question: How to physically implement the condition $V_0 = \frac{V_\pi}{2}$?

Electro-Optic modulators with KDP crystals

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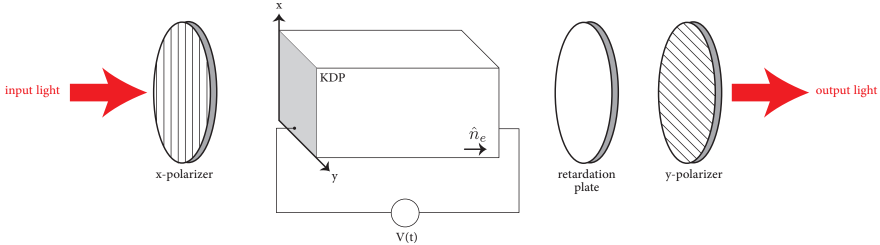
For $V_0 = \frac{V_\pi}{2}$, the phase difference $k\Delta n L = \frac{\pi}{2}$. This can be easily implemented by adding a quarter-wave retardation plate at the crystal output.



Exercise: calculate the length D along z of the retarder plate to achieve the required condition on V_0 at the output.

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- A. Yariv, *Photonics: Optical Electronics in Modern Communications* (Oxford University Press, 2006). Chapter 9
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- R. W. Boyd, *Nonlinear Optics* (Academic Press, 2008). Chapter 1 and 11