ECE325 Advanced Photonics

Light propagation in anisotropic crystals
Introduction to Plasmonics
lesson 7

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Outline

Phase modulation

Exercise: design a phase modulation system by using a KDP crystal.

Phase modulation

Exercise: design a phase modulation system by using a KDP crystal. We can use a standard amplitude modulator system, as illustrated in the previous lesson, where we launch an ordinary solution along x' or y'. In this case the output electric field reads:

$$E_{x'}(L) = E_{x'} e^{-i\frac{\omega}{c}\Delta n L} = E_{x'} e^{-i\frac{\omega n_0^3 r_{63} E_{0z} L}{2c}}. \tag{1}$$

If we modulate the voltage in time $E_{0z} = E_m \cos(\omega_m t)$, the time evolution of the electric field $e_{x'} = \Re(E_{x'})$ becomes modulated in phase:

$$e_{x'}(t) = \cos\left[\omega t - \frac{\omega L}{c}\left(n_0 - \frac{n_0^3}{2} r_{63} E_m \cos\omega_m t\right)\right]. \tag{2}$$

This system acts as a phase modulator.

Transverse modulation

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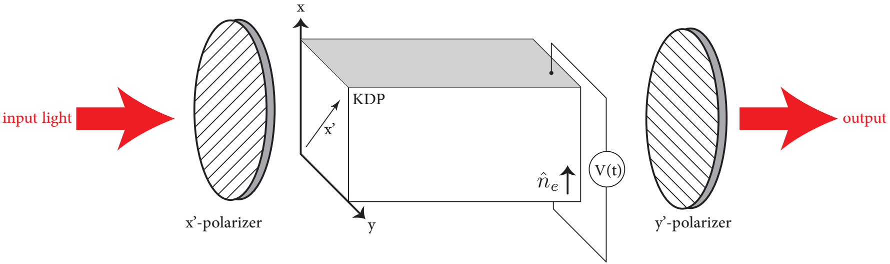


Figure: a traverse optical modulator

Transverse modulation

In the previous figure, the input light excites two ordinary waves with index n_e and $n_0 + \frac{n_0^3}{2} r_{63} E_{0z}$. The phase shift $\Delta\phi$ accumulated after a distance L in the crystal is then:

$$\Delta\phi = \frac{\omega L}{c} \left(n_0 + \frac{n_0^3}{2} r_{63} E_{0z} - n_e \right). \tag{3}$$

Exercise: Calculate the output intensity of the transverse modulator illustrated in the figure of slide 4.

Outline

Maxwell equations in lossy materials

In a non-instantaneous, linear and isotropic material, Maxwell equations read:

$$\begin{cases} \nabla \times \boldsymbol{e} = -\partial_t \boldsymbol{b}, \\ \nabla \times \boldsymbol{h} = \epsilon_0 \partial_t \boldsymbol{e} + \boldsymbol{j} \end{cases} , \quad \boldsymbol{j} = \epsilon_0 \int \mathrm{d}t\mathrm{d}\boldsymbol{r}' \sigma \left(\boldsymbol{r} - \boldsymbol{r}', t - t' \right) \boldsymbol{e}(\boldsymbol{r}, t), \tag{4}$$

with $\boldsymbol{j}$ the internal current density generated in the lossy material and σ the non instantaneous conductivity. Energy conservation is expressed by the continuity equation:

$$\frac{\partial \rho}{\partial t} + \nabla \cdot \boldsymbol{j} = 0, \tag{5}$$

being ρ the internal charge density. By combining the latter with the divergence equation $\nabla \cdot \boldsymbol{p} = -\rho$, with p the electric dipole moment per unit volume inside the material, we have:

$$\boldsymbol{j} = \frac{\partial \boldsymbol{p}}{\partial t}, \tag{6}$$

which relates the current flux with the time variation of the electric dipole moment $\boldsymbol{p}$.

Maxwell equations in lossy materials

by moving in the Fourier domain $\int dt d\boldsymbol{r} e^{i\omega t - \boldsymbol{k}\cdot r}$, we have:

$$\boldsymbol{J} = \sigma\left(\boldsymbol{K}, \omega\right) \boldsymbol{E}(\boldsymbol{k}, \omega). \qquad (7)$$

From the latter equation and (6), we have:

$$\boldsymbol{D} = \epsilon_0 \boldsymbol{E} + \boldsymbol{P} = \epsilon_0 \boldsymbol{E} + \frac{\boldsymbol{J}}{i\omega} = \epsilon_0 \left(1 + \frac{\sigma}{i\omega}\right) \boldsymbol{E} = \epsilon_0 \epsilon\left(\boldsymbol{K}, \omega\right) \boldsymbol{E}, \qquad (8)$$

with:

$$\epsilon\left(\boldsymbol{K}, \omega\right) = 1 + \frac{\sigma\left(\boldsymbol{K}, \omega\right)}{i\omega} = \epsilon_1 + i\epsilon_2, \qquad (9)$$

identifying the dielectric response of the lossy material, with real ϵ_1 and imaginary ϵ_2 contributions.

Maxwell equations in lossy materials

In optics, we usually use a complex refractive index $n + ik = \sqrt{\epsilon}$ for characterizing materials, with k the extinction coefficient. From equation (9) we have:

$$\begin{cases} \epsilon_1 = n^2 - k^2, \\ \epsilon_2 = 2nk, \end{cases} \tag{10}$$

which can be easily inverted to get n and k as a function of ϵ_1 and ϵ_2. In many situations, we can simplify the general form of the conductibility $\sigma(\boldsymbol{K}, \omega) \to \sigma(\omega)$, by considering a homogenous material. This assumption is valid as long as the wavelength λ is larger than the mean free path of the electrons in the system. This is typically true for wavelengths λ larger than the ultraviolet.

The plasma model

Is a fundamental model that describes the electrons as a free electron gas. It provides a microscopic foundation for more complex models of metals and lossy materials.

The classical motion of an electron in a plasma subjected to an external field is:

$$m\partial_t^2 \boldsymbol{r} + m\gamma\partial_t \boldsymbol{r} + q\boldsymbol{e} = 0, \tag{11}$$

being m the effective mass of the electron, n the electron density, q the electric charge and $\gamma = \frac{1}{\tau}$ the electron collision frequency, with collision time τ. For a monochromatic excitation $\boldsymbol{e} = \boldsymbol{E}e^{i\omega t}$, we can solve the equation of motion for the displacement $\boldsymbol{r}$:

$$\boldsymbol{r} = \frac{q}{m(\omega^2 + i\gamma\omega)}\boldsymbol{E}, \tag{12}$$

and the corresponding macroscopic polarization $\boldsymbol{P}$:

$$\boldsymbol{P} = -nq\boldsymbol{r} = -\frac{nq^2}{m(\omega^2 + i\gamma\omega)}\boldsymbol{E}. \tag{13}$$

The plasma model

From (6), we have:

$$\sigma(\omega) = -i\frac{\omega n q^2}{m(\omega^2 + i\gamma\omega)} = -i\omega\epsilon_0\frac{\omega_p^2}{\omega^2 + i\gamma\omega}, \quad (14)$$

with $\omega_p^2 = \frac{nq^2}{\epsilon_0 m}$ the plasma frequency of the electron gas. The corresponding dielectric constant becomes:

$$\epsilon(\omega) = 1 - \frac{\omega_p^2}{\omega^2 + i\gamma\omega}, \qquad \begin{cases} \epsilon_1(\omega) = 1 - \frac{\omega_p^2\tau^2}{1+\omega^2\tau^2}, \\ \epsilon_2(\omega) = \frac{\omega_p^2\tau}{\omega(1+\omega^2\tau^2)} \end{cases}, \quad (15)$$

the complex dielectric function of the plasma model. For typical metals, the characteristic plasma wavelength $\lambda_p = \frac{2\pi c}{\omega_p} \approx 100$ nm and the collision time is $\tau \approx 10^{-14} s$.

The undamped free electron plasma

For a collisionless plasma, we have $\gamma \approx 0$. This idealized condition leads to:

$$\epsilon(\omega) = 1 - \left(\frac{\omega_p}{\omega}\right)^2, \tag{16}$$

which represents the response of the undamped free electron plasma. Equation (16) models quite well metals at microwave ranges and at the beginning of the THz window. The reflectivity spectrum $R(\omega)$ at normal incidence for a material described by (16) is:

$$R(\omega) = \left|\frac{\sqrt{\epsilon} - 1}{\sqrt{\epsilon} + 1}\right|^2 = \left|\frac{\sqrt{1 - \frac{\omega_p^2}{\omega^2}} - 1}{\sqrt{1 - \frac{\omega_p^2}{\omega^2}} + 1}\right|^2. \tag{17}$$

For $\omega \leq \omega_p$, we have $\sqrt{\epsilon} = i\sqrt{\omega_p^2/\omega^2 - 1}$, and the system response is that of an ideal metal with $R(\omega) = 1$. For $\omega > \omega_p$, conversely, the reflectivity tends progressively to zero, indicating the presence of propagating waves in the structure.

The presence of collisions

At optical wavelengths, for $\omega\tau \ll 1$, metals are highly absorbing with $\epsilon_2 \gg \epsilon_1$:

$$\begin{cases} \epsilon_1(\omega) \approx 1 - \omega_p^2 \tau^2, \\ \epsilon_2(\omega) \approx \frac{\omega_p^2 \tau}{\omega} \end{cases} , \tag{18}$$

which leads to $n \approx k = \sqrt{\frac{\epsilon_2}{2}} = \sqrt{\frac{\omega_p \tau}{2\omega}}$. The absorption coefficient $\alpha = \frac{2k\omega}{c}$ is then:

$$\alpha = \sqrt{\frac{2\omega_p^2 \tau \omega}{c^2}}, \tag{19}$$

and the corresponding penetration length, or skin depth is $\delta = \frac{2}{\alpha} = \frac{c}{k\omega} = \sqrt{\frac{2c^2}{\omega\tau\omega_p^2}}$. In typical metals, we have $\delta \approx 100$ nm. The reflectivity spectrum shows the same behavior of the undamped case, but with a lower reflectivity $R < 1$ for $\omega < \omega_p$ due to material absorption $\epsilon_2 \neq 0$.

The contribution of interband transitions

At optical wavelengths, especially near the ultraviolet, we need to take into account interband transitions, which generate resonances in the dielectric response that are not modeled by the plasma model. The complex dielectric response of a metal is typically expressed as follows:

$$\epsilon(\omega) = \epsilon_\infty - \frac{\omega_p^2}{\omega^2 + i\gamma\omega} + \sum_n \frac{\omega_{pn}^2}{\omega^2 - \omega_{n0}^2 + 2i\gamma_n\omega}, \tag{20}$$

as a sum of the the response of a free electron plasma with a series of Lorentz oscillators describing interband transitions. Each oscillator is characterized by a resonant frequency ω_{0n}, amplitude ω_{pn} and damping γ_n. The calculation of the various coefficients in the oscillators is either done experimentally, by fitting reflectivity spectra of the material, or theoretically from first principle quantum chemistry simulations. In (20), ϵ_∞ represent the dielectric response of the system at $\omega \to \infty$. One or two Lorentz oscillators are typicaly enough to model a metal response in the visible range.

Plane waves in lossy materials

From the wave equation:

$$\nabla \times \nabla \times \boldsymbol{e} + \mu_0 \frac{\partial^2 \boldsymbol{d}}{\partial t^2} = 0, \tag{21}$$

moving into the Fourier domain $\int dt d\boldsymbol{r} e^{i\omega t - \boldsymbol{k}\cdot\boldsymbol{r}}$, we have:

$$\boldsymbol{k}(\boldsymbol{k} \cdot \boldsymbol{E}) - k^2 \boldsymbol{E} + \epsilon(\boldsymbol{K}, \omega) \frac{\omega^2}{c^2} \boldsymbol{E} = 0, \tag{22}$$

which implies the following plane wave solutions:

- Transverse waves for $\boldsymbol{k} \cdot \boldsymbol{E} = 0$. This implies the dispersion relation:

$$k^2 = \epsilon(\boldsymbol{k}, \omega) \frac{\omega^2}{c^2}, \tag{23}$$

- Longitudinal waves for $\boldsymbol{k} \times \boldsymbol{E} = 0$. From (22), we have:

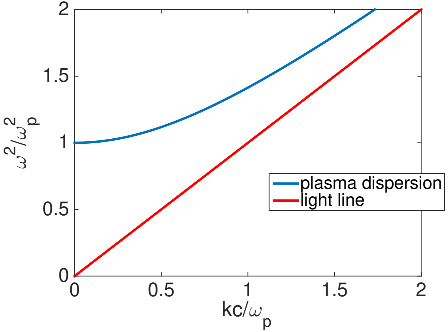
$$\epsilon(\boldsymbol{k}, \omega) = 0. \tag{24}$$

This shows that longitudinal waves do not exist at every frequency, but only at the zeros of $\epsilon(\boldsymbol{k}, \omega)$.

Transverse Plane waves: volume plasmons

For the sake of simplicity and without loss of generality, we consider the dielectric permittivity $\epsilon(\omega) = 1 - \frac{\omega_p^2}{\omega^2}$ of a collisionless plasma (see Eq. 16 of lesson 7). This yield the following dispersion relation for longitudinal waves:

$$\omega = \omega_p \sqrt{1 + \frac{k^2 c^2}{\omega_p^2}}. \qquad (25)$$



Longitudinal plane waves exist only above the plasma frequency for $\omega \geq \omega_p$. These plane waves are called volume plasmons.

Exercise: calculate the dispersion relation in the presence of nonzero collision time τ in the plasma model.

Outline

Reference texts

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