ECE325 Advanced Photonics

Waveguide theory
lesson 9

Andrea Fratalocchi

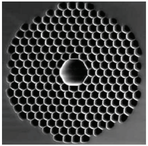
www.primalight.org

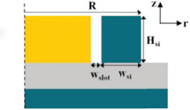
July 3, 2021

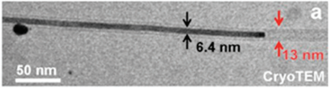
Outline

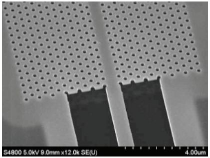
Optical waveguides

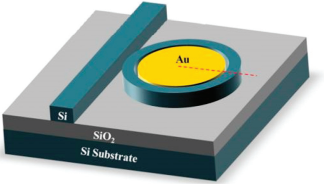
Beyond elementary slab dielectric structures, optical waveguides can encompass complex 2D geometries and materials including dielectric, metals or a combination of them, in both long hauls and integrated configurations.

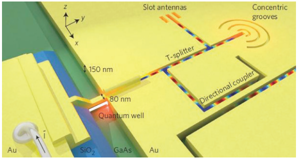












Outline

Theory of waveguide modes

A waveguide mode is a propagating wave solution inside a waveguide structure with the following form:

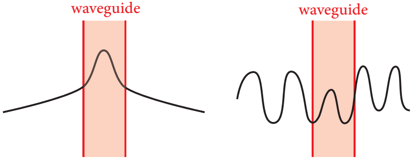
$$\begin{bmatrix} \boldsymbol{e}(\boldsymbol{r}, t) \\ \boldsymbol{h}(\boldsymbol{r}, t) \end{bmatrix} = \begin{bmatrix} \boldsymbol{E}_\nu(x, y) \\ \boldsymbol{H}_\nu(x, y) \end{bmatrix} e^{i\omega t - i\beta_\nu z}, \tag{1}$$

characterized by vectors $\boldsymbol{E}_\nu$, $\boldsymbol{H}_\nu$, which describes the spatial distribution of the mode in the plane (x, y) orthogonal to the direction of propagation z, and the propagation constant β_ν. The index ν, which identify the particular mode, can be discrete or continuous, typically depending on the behavior of the mode at $x \to \infty$ and $y \to \infty$. The amplitude of the mode is constant along the propagation direction z.

General theory of waveguide modes

For general waveguide structures, there are two different types of modes:

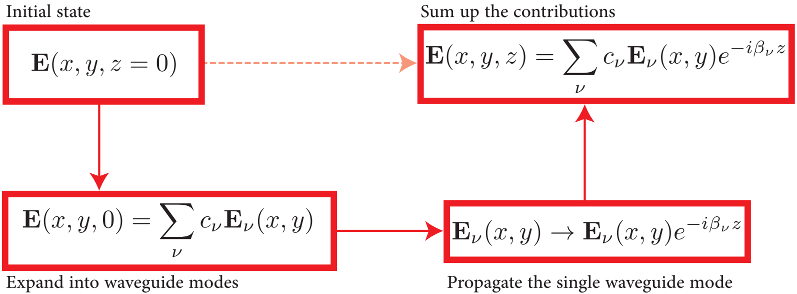
- Guided modes. These modes are characterized by a transverse amplitude that tends to zero outside the waveguide: $\boldsymbol{E}_\nu(x,y) \to 0$ and $\boldsymbol{H}_\nu(x,y) \to 0$ for $(x,y) \to \infty$. The mode profile is localized inside the waveguide, allowing for a guided propagation of energy inside the structure, and the index ν is discrete. These are the modes computed by the ray theory in the slab case in the previous lesson.
- Radiation modes. This class of modes possess transverse amplitudes $\boldsymbol{E}_\nu(x,y)$ and $\boldsymbol{H}_\nu(x,y)$ periodically oscillating inside and outside the waveguide, and are periodic at $x, y \to \infty$. The mode profile is not localized in any region of the space, and the index ν is continuous.



a guided mode   a radiation mode

General theory of waveguide modes

It can be demonstrated from the hermitian nature of Maxwell equations under vanishing or periodic boundary conditions at infinity, that the set of modes (guided + radiation) is complete and can express all possible waves propagating in the waveguide. This implies we can use a familiar scheme for propagating fields inside optical waveguides:



Outline

Mode normalization and orthogonality

From Maxwell equations:

$$\begin{cases} \nabla \times \boldsymbol{E} = -i\omega\mu\boldsymbol{H}, \\ \nabla \times \boldsymbol{H} = i\omega\epsilon(\omega)\boldsymbol{E}, \end{cases} \tag{2}$$

if $\boldsymbol{E}_1$, $\boldsymbol{H}_1$ and $\boldsymbol{E}_2$, $\boldsymbol{H}_2$ identify two different solutions, an important identity is the following expression, which we leave the demonstration as an exercise:

Reciprocity theorem

$$\nabla\left(\boldsymbol{E}_1 \times \boldsymbol{H}_2^* + \boldsymbol{E}_2^* \times \boldsymbol{H}_1\right) = 0. \tag{3}$$

This constitutes the reciprocity theorem of Maxwell equations. Equation (3) holds for any set of solutions. If we consider $\boldsymbol{E}_i$, $\boldsymbol{H}_i$ $(i = \nu, \mu)$ to be a set of waveguide modes, we then have:

$$\left[\nabla_\perp - i\left(\beta_\nu - \beta_\mu\right)\hat{z}\right]\left(\boldsymbol{E}_\nu \times \boldsymbol{H}_\mu^* + \boldsymbol{E}_\mu^* \times \boldsymbol{H}_\nu\right) = 0, \tag{4}$$

Mode normalization and orthogonality

Equation (4) has the expression of a conservation law, written in divergence free form. By integrating along the plane (x, y) each member of the equation, we obtain the following condition:

$$i\left(\beta_\nu - \beta_\mu\right) \iint_{-\infty}^{\infty} dxdy \left(\boldsymbol{P}_{\nu\mu} \cdot \hat{z}\right) = \iint_{-\infty}^{\infty} dxdy \nabla_\perp \boldsymbol{P}_{\nu\mu} = 0, \tag{5}$$

where $\boldsymbol{P}_{\nu\mu} \equiv \boldsymbol{E}_\nu \times \boldsymbol{H}_\mu^* + \boldsymbol{E}_\mu^* \times \boldsymbol{H}_\nu$ and the last equality $\iint dxdy \nabla_\perp \boldsymbol{P}_{\nu\mu} = 0$ holds for periodic or constant boundary conditions at infinity $(x, y) \to \infty$. Equation (5) implies the following orthogonality condition among the modes:

Orthonormality condition

$$\frac{1}{4} \iint_{-\infty}^{\infty} dxdy \left(\boldsymbol{P}_{\nu\mu} \cdot \hat{z}\right) = \delta_{\mu\nu} \tag{6}$$

which is usually normalized with the factor $\frac{1}{4}$.

Mode normalization and orthogonality

Advanced question: why the factor $\frac{1}{4}$ in the orthogonality condition and what is the physical meaning of Eq. (6)?

Outline

Reference texts

- T. Tamir, *Guided-wave optoelectronics* (Springer, 1988). Chapter 2.
- H. Nishihara, *Optical Integrated Circuits* (McGraw Hill, 1989). Chapters 2 & 3.
- K. Okamoto, *Fundamentals of Optical Waveguides* (Academic Press, 2010). Chapter 1.
- C. A. Balanis, *Advanced Engineering Electromagnetics* (Wiley, 1989). Chapter 7.