ECE325 Advanced Photonics

Light propagation in anisotropic crystals
lesson 5

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Outline

Refraction of light in anisotropic crystals

To calculate the wavevector direction and the refractive index of the plane waves refracted at an interface with an anisotropic medium, we impose phase-matching conditions at the interface separating the two different materials. Recalling that the phase of the plane wave is $i(\omega t - \boldsymbol{k} \cdot \boldsymbol{r})$, and assuming that the incident wavevector forms an angle θ with the plane normal and the surface of separation oriented toward x, the phase-matching condition reads:

$$i\omega t - i\frac{\omega}{c} n_{in} \sin\theta_{in} \cdot x = i\omega t - i\frac{\omega}{c} n(\hat{s}) \sin\alpha \cdot x, \tag{1}$$

with n_{in} and θ_{in} the refractive index of the isotropic material and input angle of incidence of light, respectively. Equation (1) leads to:

Snell law for anisotropic crystals

$$n_{in} \sin\theta_{in} = n(\hat{s}) \sin\alpha$$

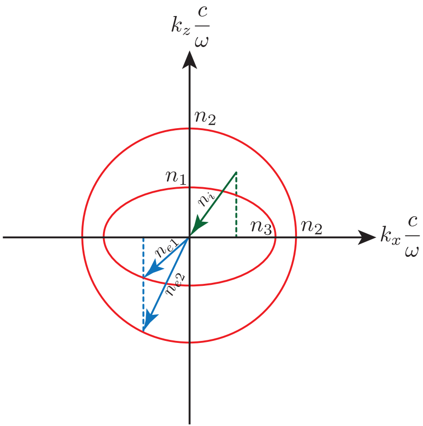
In the right hand side this equation, both α and $n(\hat{s})$ are unknown to be found, contrary to an isotropic material where n is given and independent on $\hat{s}$.

Refraction of light in anisotropic crystals

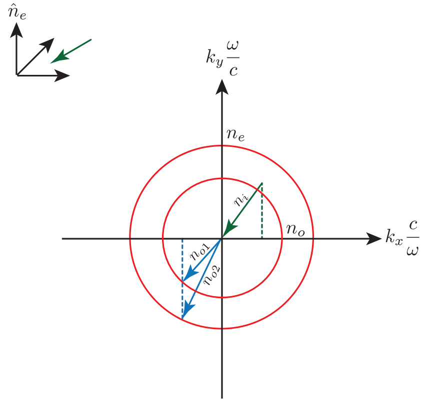
To calculate $n(\hat{s})$ the angle α formed by the wavevector of the refracted wave $\hat{s}$ and the plane normal, we use the dispersion relation of the crystal:

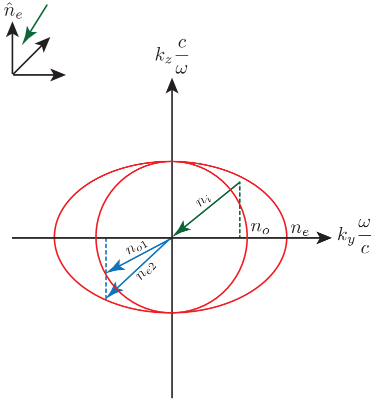
$$|\boldsymbol{k}| = \frac{\omega}{c} n(\hat{s}), \tag{2}$$

which defines the surfaces of constant phase for each direction of the wavevector $\hat{s}$. By applying Snell law and by using the dispersion diagram to match the field phase at the interface, we obtain the wavevector directions and the refractive indices of the plane waves refracted in the anisotropic crystal.



Analysis Uniaxial crystals

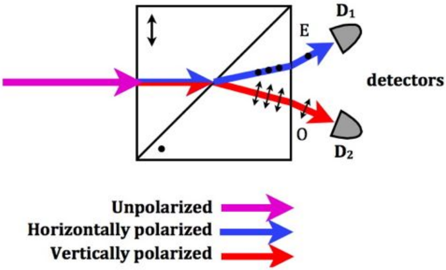




This analysis is for a positive Uniaxial crystal, with $n_e > n_o$. In the left case, the refraction is composed by two ordinary waves, with indices n_{o1} and n_{o2}, while in the right case we have one ordinary wave and one extraordinary wave.

Wollaston Prism (WP) polarizer beam splitter

A WP consists of two prisms of a Uniaxial crystal, made of calcite, with orthogonal optical axis, whose direction is reported in the figure. Light refraction at the second interface, following Snell law, originates ordinary and extraordinary waves that propagates at different angles. Light emerging from the WP is then split according to its polarization state.



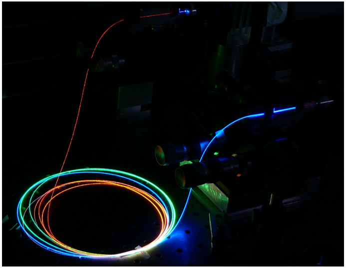
Outline

Elements of nonlinear optics

For a generic nonlinear material, the polarization response $\boldsymbol{p}$ is no longer linear, but it becomes (assuming an instantaneous response):

$$\begin{aligned}\boldsymbol{p} = \epsilon_0 \underline{\underline{\chi}}(\boldsymbol{e}) =& \epsilon_0 \underline{\underline{\chi}}\boldsymbol{e} + \epsilon_0 \underline{\underline{\chi}}^{(2)} \boldsymbol{e}\cdot\boldsymbol{e} + \epsilon_0 \underline{\underline{\chi}}^{(3)} \boldsymbol{e}\cdot\boldsymbol{e}\cdot\boldsymbol{e} + O(\boldsymbol{e}^4) = \\ & \boldsymbol{p}^{(0)} + \boldsymbol{p}^{(1)} + \boldsymbol{p}^{(2)} + \boldsymbol{p}^{(3)} + O(\boldsymbol{e}^4), \\ \boldsymbol{p}^{(i)} = \epsilon_0 \underline{\underline{\chi}}^{(i)} \boldsymbol{e}^i, & \end{aligned} \tag{3}$$

with $\underline{\underline{\chi}}$ the linear susceptibility response of the material and $\underline{\underline{\chi}}^{(i)}$ the nonlinear optical susceptibility of order i. Nonlinear high-order terms mix all the components of the electric field, allowing to observe complex forms of light-matter interaction.



Second order interactions

Let us consider the effects associated to a nonzero second order nonlinear susceptibility:

$$p_h^{(2)} = \epsilon_0 \sum_{jk} \chi_{hjk} e_j e_k \equiv \epsilon_0 \chi_{hjk} e_j e_k, \tag{4}$$

where in the last expression we used the Einstein convention over repeated indices. We consider an electric field composed by a traveling wave at frequency ω and a static field $\boldsymbol{E}_0$:

$$\boldsymbol{e} = \frac{1}{2}\boldsymbol{E}_\omega e^{i\omega t} + \frac{1}{2}\boldsymbol{E}_\omega^* e^{-i\omega t} + \boldsymbol{E}_0, \tag{5}$$

By substituting (5) into (4), we have

$$\begin{aligned} p_h^{(2)} =& \epsilon_0 \chi_{hjk} \left(\frac{1}{2} E_{\omega j} e^{i\omega t} + \frac{1}{2} E_{\omega j}^* e^{-i\omega t} + E_{0j} \right) \\ & \times \left(\frac{1}{2} E_{\omega k} e^{i\omega t} + \frac{1}{2} E_{\omega k}^* e^{-i\omega t} + E_{0k} \right), \end{aligned} \tag{6}$$

Second order interactions

After some straightforward algebra, we obtain:

$$p_h^{(2)} = \epsilon_0 \frac{\chi_{hjk}}{4} \cdot \left(E_{\omega j} E_{\omega k} e^{i2\omega t} + E^*_{\omega j} E^*_{\omega k} e^{-i2\omega t} \right) + \tag{7}$$

$$\frac{\epsilon_0}{4} \cdot E_{0k} \left(\chi_{hjk} + \chi_{hkj} \right) \left(E_{\omega j} e^{i\omega t} + E^*_{\omega j} e^{-i\omega t} \right) + \tag{8}$$

$$\epsilon_0 \frac{\chi_{hjk}}{4} \left(E_{\omega j} E^*_{\omega_k} + E_{\omega k} E^*_{\omega_j} + E_{0j} E_{0k} \right) \tag{9}$$

The first term of the nonlinear polarization $\boldsymbol{p}^{(2)}$ oscillates at 2ω and is a nonlinear effect known as second harmonic generation. The second term oscillates at the input frequency ω and is known as electro-optic effect, while the last term oscillates at zero frequency and is known as optical rectification. Through nonlinear light-matter interactions, we can generate a complex electromagnetic field composed by different frequency components.

The Electro-Optic effect

In the second order nonlinear polarization response $p^{(2)}$, the term oscillating at ω sums up with the linear susceptibility of the material, proving an additional contribution to the material response at the frequency of the input wave:

$$p_h^{(1)} = \frac{\epsilon_0}{2}\left(\chi_{hj} + \Delta\chi_{hj}\right) E_{\omega j} e^{i\omega t} + \mathrm{c.c.}, \tag{10}$$

with $\Delta\chi_{hj} = E_{0k}\left(\chi_{hjk} + \chi_{hkj}\right)$. The components of the dielectric permittivity tensor of the material then become $\epsilon_{hj} = 1 + \chi_{hj} + \Delta\chi_{hj}$, and the corresponding refractive indices:

$$n_{hj} = \sqrt{1 + \chi_{hj} + \Delta\chi_{hj}} = \tilde{n}_{hj} + \Delta n_{hj} = \tilde{n}_{hj} + \frac{1}{2\tilde{n}_{hj}}\Delta\chi_{hj} + O\left(\Delta\chi_{hj}^2\right), \tag{11}$$

being $\tilde{n}_{hj} = \sqrt{1 + \chi_{hj}}$ the linear refractive index and Δn_{hj} the refractive index shift.

The Electro-Optic effect

The refractive index shift reads as follows:

$$\Delta n_{hj} = \frac{(\chi_{hjk} + \chi_{hkj})}{2\tilde{n}_{hj}} E_{0k}, \tag{12}$$

and depend linearly on the electrostatic field applied E_{0k}.
Thanks to this specific nonlinear light-matter interaction, we can change the refractive of a second order nonlinear material by applying an external static electric field $\boldsymbol{E}_0$.

Electro-Optic effect The change of the optical properties of a material in response to a static electric field, or a field that varies slowly compared to light frequency.

Question: is the Electro-Optic effect a linear or a nonlinear effect?

The Electro-Optic effect: some general considerations

The Electro-Optic effect is the result of a second order nonlinear light-matter interaction, however, the index variation described by (12) is linear in the static electric field applied E_{0k}. For this reason, this specific light-matter interaction is called linear Electric-Optic (EO) effect. By exploiting different nonlinear interactions, we can also observe quadratic EO effects, which depend on E_{0k}^2.
The Electro-Optic effect can be observed in crystals that have a non vanishing second order nonlinear optical susceptibility. A necessary condition is the lack of some specific symmetries in the crystal structure. Some isotropic materials, with $\tilde{n}_{hj} = \tilde{n} = \sqrt{1+\chi}$, have a non-centrosymmetric structure that allows to observe a nonvanishing electro-optic effect. In this cases, the EO effect turns the material into an anisotropic system. An example of this material is *GaAs*, an isotropic medium that belongs to the $\bar{4}3m$ symmetry group and has a nonvanishing second order nonlinear response.

Outline

Reference texts

- A. Yariv, *Photonics: Optical Electronics in Modern Communications* (Oxford University Press, 2006). Chapter 9
- A. Yariv, P. Yeh, *Optical Waves in Crystals*, (Wiley-Interscience, 2002). Chapter 4.
- R. W. Boyd, *Nonlinear Optics* (Academic Press, 2008). Chapter 1 and 11
- GaAs Electro-Optic properties can be found at http://onlinelibrary.wiley.com/doi/10.1002/9781119083405.app7/pdf