ECE325 Advanced Photonics

Directional coupling
lesson 13

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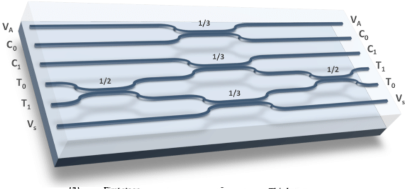
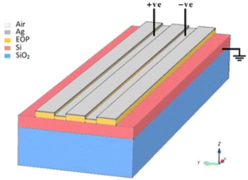
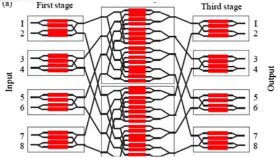
Outline

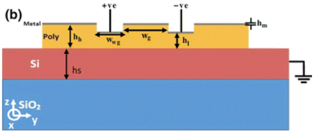
Directional coupling

Directional coupling encompasses a large class of CMT based devices, designed when modes propagates in the same direction z. These systems form modular components for integrated light switches, quantum simulators, quantum computers, optical neurocomputers, and integrated light modulators.



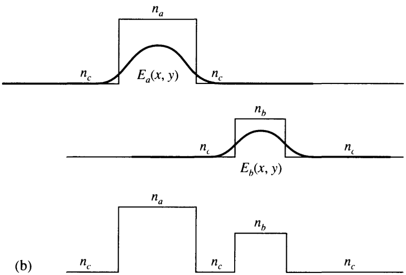






The basic architecture: directional coupler

The directional coupler represents the basic architecture used in all of these systems.



It is composed of two waveguides separated by a finite distance. In the following analysis we will assume for simplicity that the waveguides are monomodal and support a fundamental TE_0 modes, whose electric field E_a and E_b is sketched in the figure on the left.

The dielectric permittivity of the system $\epsilon(x, y)$ is decomposed as follows:

$$\epsilon(x, y) = \epsilon_1(x, y) + \epsilon_2(x, y), \quad (1)$$

with $\epsilon_i(x, y)$ the permittivity of the waveguide $i = 1, 2$.

The basic architecture: directional coupler

In order to apply the CMT, we expand the total electromagnetic field as the superposition of the fundamental modes of the two waveguides:

$$\begin{bmatrix} \boldsymbol{E}_t(x,y,z) \\ \boldsymbol{H}_t(x,y,z) \end{bmatrix} = a_1(z) \begin{bmatrix} \boldsymbol{\mathcal{E}}_1(x,y) \\ \boldsymbol{\mathcal{H}}_1(x,y) \end{bmatrix} e^{-i\beta_1 z} + a_2(z) \begin{bmatrix} \boldsymbol{\mathcal{E}}_2(x,y) \\ \boldsymbol{\mathcal{H}}_2(x,y) \end{bmatrix} e^{-i\beta_2 z}. \quad (2)$$

We then apply the reciprocity condition at Lesson 11, slide 6 by assuming:

1. A canonical structure composed by the first waveguide $\epsilon' = \epsilon_1$, perturbation $\Delta\epsilon = \epsilon_2$ represented by the second waveguide, and project over the fundamental mode of the canonical structure $\boldsymbol{\mathcal{E}}_1$ and $\boldsymbol{\mathcal{H}}_1$.
2. A canonical structure composed by the second waveguide $\epsilon' = \epsilon_2$, perturbation $\Delta\epsilon = \epsilon_1$ represented by the first waveguide, and project over the fundamental mode of the canonical structure $\boldsymbol{\mathcal{E}}_2$ and $\boldsymbol{\mathcal{H}}_2$.

The basic architecture: directional coupler

This approach yields the following set of CMT equations describing propagation of light inside this structure:

$$\partial_z \begin{bmatrix} a_1(z) \\ a_2(z) \end{bmatrix} = -i \begin{bmatrix} C_{11} & C_{21} e^{-i\Delta\beta z} \\ C_{12} e^{i\Delta\beta z} & C_{22} \end{bmatrix} \cdot \begin{bmatrix} a_1(z) \\ a_2(z) \end{bmatrix} \tag{3}$$

with $\Delta\beta = \beta_2 - \beta_1$ and:

$$C_{ij} = \frac{\omega\epsilon_0}{4} \iint dxdy \epsilon_j \left(\boldsymbol{\mathcal{E}}_i^* \boldsymbol{\mathcal{E}}_j + \frac{\epsilon_i}{\epsilon_i + \epsilon_j} \mathcal{E}_{zi}^* \mathcal{E}_{zj} \right) \quad i \neq j, \tag{4}$$

$$C_{11} = \frac{\omega\epsilon_0}{4} \iint dxdy \epsilon_2 \left(|\boldsymbol{\mathcal{E}}_1|^2 + \frac{\epsilon_1}{\epsilon_1 + \epsilon_2} |\mathcal{E}_{z1}|^2 \right), \tag{5}$$

$$C_{22} = \frac{\omega\epsilon_0}{4} \iint dxdy \epsilon_1 \left(|\boldsymbol{\mathcal{E}}_2|^2 + \frac{\epsilon_2}{\epsilon_2 + \epsilon_1} |\mathcal{E}_{z2}|^2 \right), \tag{6}$$

The conservation of energy demands that $C_{12} = C_{21}^*$.

The basic architecture: directional coupler

We then make the following coordinate transform:

$$\begin{cases} a_1 = b_1 e^{-iC_{11}z}, \\ a_2 = b_2 e^{-iC_{22}z} \end{cases} \tag{7}$$

which leads to:

$$\partial_z \begin{bmatrix} b_1(z) \\ b_2(z) \end{bmatrix} = -i \begin{bmatrix} 0 & \delta \\ \delta^* & 0 \end{bmatrix} \cdot \begin{bmatrix} b_1(z) \\ b_2(z) \end{bmatrix} \tag{8}$$

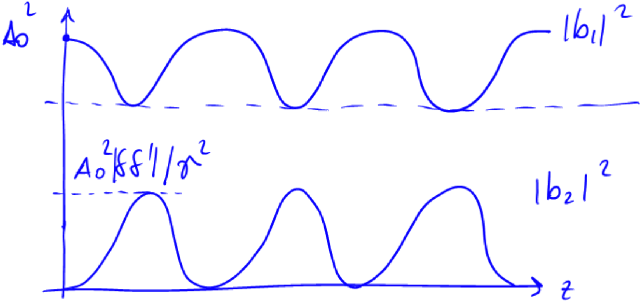
with $\delta = C_{12}e^{-i(\Delta\beta + C_{11} - C_{22})z} = C_{12}e^{-i\Delta z}$. The solution of (8) is easily found by assuming an initial condition with one energy inside one waveguide:

$$\begin{cases} b_1(0) = A_0, \\ b_2(0) = 0 \end{cases}, \begin{cases} b_1(z) = A_0 e^{i\frac{\Delta}{2}z} \left[\cos(\gamma z) - i\frac{\Delta}{2\gamma}\sin(\gamma z)\right], \\ b_2(z) = A_0 e^{-i\frac{\Delta}{2}z} \frac{|\delta|}{i\gamma}\sin(\gamma z) \end{cases} \tag{9}$$

with $\gamma^2 = \left(\frac{\Delta}{2}\right)^2 + |\delta|^2$.

The basic architecture: directional coupler

The resulting dynamics is that of a periodic energy transfer between the arms of the two waveguides, with period $z_T = \frac{2\pi}{\gamma}$.

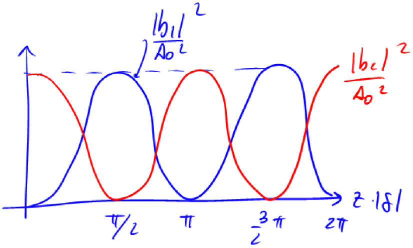


The maximum power transferred on the waveguide 2 is $P_m = \frac{|\delta|^2}{\gamma^2}$.

The basic architecture: directional coupler

The maximum transfer of power occurs in phase-matching conditions: $\Delta = 0$, observed when the two waveguides are identical. In this condition the evolution becomes:

$$\begin{cases} |b_1(z)|^2 = A_0^2 \cos(|\delta| z)^2, \\ |b_2(z)|^2 = A_0^2 \sin(|\delta| z)^2, \end{cases} \quad (10)$$



which describes a periodic full power exchange with distance $L_c = \frac{\pi}{2|\delta|}$ called coupling distance.

Outline

Universality of the directional coupler

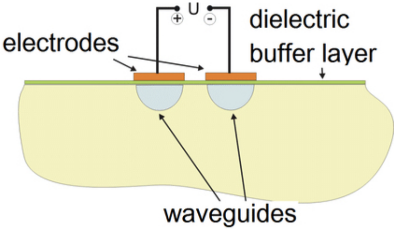
The directional coupler is the most general physical system describing interactions between two states, and as such it provides a universal dynamics that is routinely observed in many scientific areas.
An important examples is in quantum mechanics (QM). The QM equivalent to the directional couple is a system of two quantum wells with potentials depth V_1 and V_2. It can be demonstrated that Maxwell equations can be mapped to the Schrödinger equation describing the dynamics of electron wavefunctions if we apply the following substitutions:

1. the optical propagation axis z becomes the QM time t.
2. the refractive index $n(x)$ becomes a QM negative potential $-V(x)$.

Within this representation, optical waveguides modes become quantum-mechanical bound states. Directional coupling is manifested as a periodic wavefunction oscillation between the two quantum wells. This analogy is useful to understand many other analogy between Photonics and QM phenomena, and also exploited experimentally to observe QM effects within optical setups.

Outline

Ultra-fast light switches and multiplexers



The main idea is to use control the refractive index of one arm with a nonlinear effect such as, e.g., the electro-optic effect. This, in turn, will adjust the propagation constant β of one mode controllably, allowing fine tune of the resulting mismatch term Δ.

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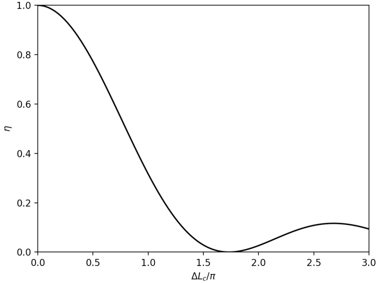
If we start from a configuration with two waveguides supporting two identical modes propagation constants $\beta_1 = \beta_2$, and choose $L_c = \frac{L}{2} = \frac{\pi}{2|\delta|}$, all the input power launched in one waveguide at $z = 0$ is transferred to the other one at $z = L_c$. This initial configuration is called cross-state (all power is cross-transferred from one arm to the other).

Ultra-fast light switches

When $\Delta \neq 0$, conversely, we have:

$$\eta = \frac{|b_2(L_c)|^2}{|b_1(0)|^2} = \left(\frac{\pi}{2}\right)^2 \operatorname{sinc}\left[\frac{1}{2}\sqrt{1 + \frac{\Delta^2 L_c^2}{\pi^2}}\right], \tag{11}$$

with η the coupling efficiency and $\operatorname{sinc}(x) = \sin(\pi x)/\pi x$.



By adjusting the mismatch Δ to $\Delta = \frac{\pi\sqrt{3}}{L_c}$, we inhibit transfer of energy to the other waveguide and the device maintains the output on the same channel (bar-state). By piloting the device from $\Delta = 0$ (cross-state) to $\Delta = \frac{\pi\sqrt{3}}{L_c}$ (bar-state), the coupler operates as an integrated switch.

Ultra-fast light multiplexers

They employ the same scheme as before, controlling the device with a periodic drive that multiplexes signal staggered in time and travelling on different arms at the input into a single arm at the output.

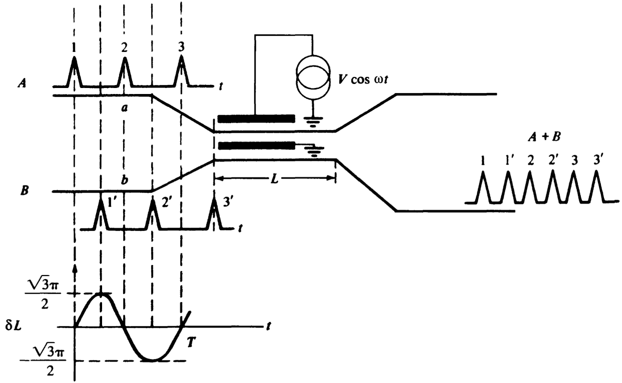


Figure: In this figure the δ used corresponds to $\frac{\Delta}{2}$ in our notation.

Outline

Reference texts

- A. Yariv, P. Yeh, *Photonics: optical electronics in modern communications* (Oxford press, 2007), Chap. 13