## ECE325 Advanced Photonics Other applications of directional coupling lesson 14

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ECE325 Advanced Photonics

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### Trasverse couplers

## 2 Side couplers

### 3 Prism coupling

#### Grating coupling

• Application of grating coupler to metrology

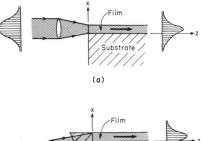
#### 5 Reference texts

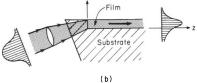
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# Trasverse coupling

In this type of *direct* or *head-on* coupler, light is injected in the waveguide as illustrated in the figure on the right side. The conversion is achieved, as explained in the lesson 10, by matching the transverse profile of the input field with the desired mode. If any mismatch occurs between the incident beam profile and the shape of the waveguide mode, electromagnetic energy is lost into radiation modes. In practical application, the mode matching can never be exact and these schemes, though easy to implement, are not the best solutions.





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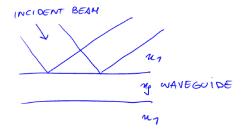
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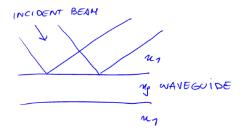
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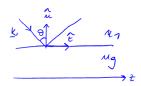
Question: would it be possible to couple electromagnetic energy inside a waveguide by the scheme illustrated in the figure below?



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The incident beam has to satisfy boundary conditions of Maxwell equations.

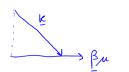


The tangential component  $\boldsymbol{E}_{t} = \hat{t} \cdot \boldsymbol{E}$  is conserved. In the free space:  $|\boldsymbol{E}_{t}| \sim e^{-ik\sin\theta z}$ , In the waveguide:  $|\boldsymbol{E}_{t}| \sim e^{-i\beta_{\mu}z}$ , with  $k = 2\pi n_{1}/\lambda$  and  $\beta_{\mu} = 2\pi n_{eff}/\lambda$ .

Phase-matching conditions between the impinging wave and the waveguide mode implies:

$$n_1 \sin \theta = n_{eff}.$$
 (1)

However, for a waveguide mode it is always  $n_{eff} > n_1$ , and as such, phasematching is never happening. This implies in this form of coupling, there is zero energy flowing into the waveguide.



The figure on the left provides a graphical picture to this type of interaction. For every guided mode  $\mu$ , we have  $|\mathbf{k}| < \beta_{\mu}$ : the free-space vector component is too *short*, and phase-matching cannot be implemented efficiently.

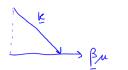
How to address this problem?

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How to address this problem? We need to increase the wavevector of photons travelling in the free-space. In the following, we will illustrate two methods for accomplishing this task.

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### Trasverse couplers

## 2 Side couplers



#### Grating coupling

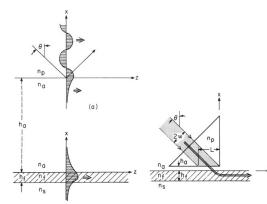
• Application of grating coupler to metrology

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# Prism assisted waveguide coupling



If we employ a prism with different refractive index  $n_p \ge n_1$ , we now have the following phase-matching condition:

$$n_p \sin \theta = n_{eff}$$
,

which can be achieved for a given  $n_p$  by entering the prism at a specific angle  $\theta$ .

Typically, the index  $n_p$  is chosen so that the angle  $\theta$  is higher than the critical angle for total internal reflection, creating an interaction between the waveguide mode and the incident field in which the two exchange energy via evanescent tails. If the two are phase-matched, exponential energy buildup occurs in the waveguide mode.

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## CMT analysis

The CMT model of this interaction is the same of the directional coupler, as it is based on two modes exchanging power in two different structures:

$$\frac{d}{dz} \begin{bmatrix} a_{prism}(z) \\ a_{guide}(z) \end{bmatrix} = -i \begin{bmatrix} C_{00} & \delta \\ \delta^* & C_{11} \end{bmatrix} \cdot \begin{bmatrix} a_{prism}(z) \\ a_{guide}(z) \end{bmatrix},$$
(2)

with  $\delta = C_{12}e^{-i\Delta\beta z}$ . The phasematching condition  $kn_p \sin \theta = \beta_{\mu}$  imposes  $\Delta\beta = 0$ , obtaining the same evolution dynamics of the directional coupler, with complete transfer of energy after one coupling length  $L_c = \frac{\pi}{2C_{12}}$ .

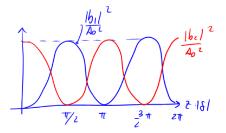


Figure: 
$$b_1 = a_{guide}, b_2 = a_{prism}$$

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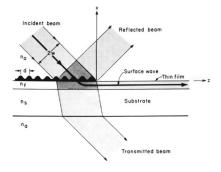
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The main issue of using a prism is that it is not a component that can be integrated. A better solution is offered by a grating structure, characterized by a periodic perturbation of permittivity  $\Delta(x, y, z) = \Delta(x, y, z + \Lambda)$  of period  $\Lambda$ .



We have:

$$\epsilon(x, y, z) = \epsilon'(x, y) + \Delta \epsilon(x, y, z).$$

By expanding the perturbation term  $\Delta \epsilon$  with a Fourier series:

$$\Delta \epsilon(x, y, z) = \sum_{m} \epsilon_{m}(x, y) e^{i\frac{2\pi m}{\Lambda}z}.$$
 (3)

We now consider the effect of this periodic perturbation on the linear polarization response  $\mathbf{P} = \epsilon_0 \epsilon \cdot \mathbf{E}$  of a generic material.

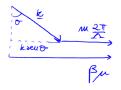
We have:

$$\boldsymbol{P} = \epsilon_0 \epsilon \boldsymbol{E} = \epsilon_0 (\epsilon' + \Delta \epsilon) \boldsymbol{E} = \epsilon' \boldsymbol{E} + \sum_m \epsilon_m e^{i \frac{2\pi m}{\Lambda} z} \boldsymbol{E}.$$
 (4)

The last terms defined a new electric field emerging from the material, composed of an infinite number of waves (diffracting orders), each with propagation constant  $2\pi m\Lambda$ . Thanks to such response induced by the grating structure, it is now possible to perform phase-matching with one diffractive component of the grating:

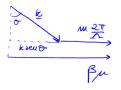


The phase-matching now reads as:  $k_z + m \frac{2\pi}{\Lambda} = \beta_m$ , with  $k_z$  the impinging wavevector along z,  $\beta_m$  the propagation constant of the guided mode and  $m \frac{2\pi}{\Lambda}$  the contribution arising from the grating.



The figure on the left provide a graphical representation to the phase-matching condition. Thanks to the grating, we can match the propagation constant of any waveguide mode just by simply creating a periodic structure with the required period, which can be changed continuously across a wide range of values. The CMT analysis of the grating is exactly the same as the prism.

What is the impact of using a different grating profile on the coupling dynamics?



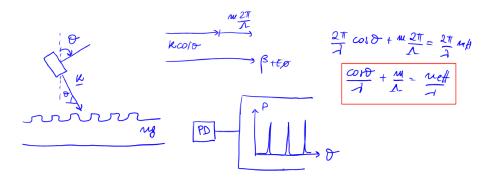
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What is the impact of using a different grating profile on the coupling dynamics? It changes the coupling distance  $L_c = \frac{\pi}{2C_{12}}$ , which depends on the coupling coefficient  $C_{12}$  that, in turns, depends on the grating profile  $\Delta \epsilon$ .

Which diffractive order is in general better to use?

Which diffractive order is in general better to use? The one that contains the strongest amplitude  $\epsilon_m$ . Typically, this occurs for m = 1 or the order in which the grating is designed to scatter the largest amplitude. Can you design an application of this coupler in the field of metrology?

# Measurement of effective index of guided modes



From the phase-matching condition:

$$\frac{\cos\theta}{\lambda} + \frac{m}{\Lambda} - \frac{n_{eff}}{\lambda} = 0,$$
(5)

by knowing the wavelength  $\lambda$  and the period  $\Lambda$  of the grating, we can scan over  $\theta$  and measure with high precision (up to 4 digits) the refractive index  $n_{eff}$  of the guided modes of a general waveguide.

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### • T. Tamir, Integrated Optics (Springer, 1975), Chap. 3

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