ECE325 Advanced Photonics

Distributed structures with feedback
lesson 15

Andrea Fratalocchi

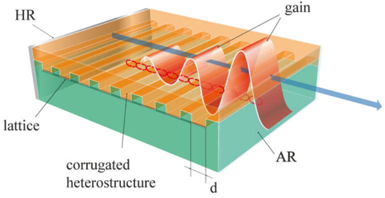
www.primalight.org

April 18, 2022

Outline

Distributed feedback

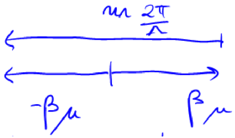
In the previous lesson, we have discussed how a periodic index modulation can couple incoming radiation with a guided mode that is propagating in the same direction thanks to the grating contribution $m\frac{2\pi}{\Lambda}$, with m the diffracting order considered.



In any grating however, any scattering order m exist with positive and negative sign: $\pm m$. This result relies on the fact that the grating index perturbation $\Delta\epsilon$ is real, and the Fourier series contains complex conjugated orders $\pm m$ for any grating profile that a user can implement.

Distributed feedback

This condition makes it possible to use a grating structure to couple energy from one mode to a backward propagating one. A typical scenario, referred as Bragg coupling is the following:



in which the grating is used to coupled a single mode of one waveguide to same mode propagating backward. Phase-matching implies that:

$$m\frac{2\pi}{\Lambda} = 2\beta_\mu, \tag{1}$$

which is known as Bragg condition. In this case, the grating is used to implement a modal reflector.

CMT analysis

Let us model this interaction in the general case by using CMT theory. The analysis is the same for the directional coupler, with the only difference that we need to consider a negative sign in the mode backward propagating due to the different direction of the Pointing vector:

$$\frac{d}{dz}\begin{bmatrix} a_+(z) \\ -a_-(z) \end{bmatrix} = -i \begin{bmatrix} C_{11} & C_{12}e^{-i\Delta\beta z} \\ C_{12}^* e^{i\Delta\beta z} & C_{22} \end{bmatrix} \cdot \begin{bmatrix} a_+(z) \\ a_-(z) \end{bmatrix}, \tag{2}$$

with $|a_\pm|^2$ the power carried out by the forward (+) or backward (-) guided mode, $\Delta\beta = \beta_+ + \beta_- - m\frac{2\pi}{\Lambda}$ the mismatch respect to the Bragg condition, C_{ii} $(i = 1, 2)$ self-overlap terms and C_{12} overlap integrals between the two modes created by the grating:

$$C_{12} = \frac{\omega\epsilon_0}{4}\int dxdy\boldsymbol{E}_+\epsilon_m\boldsymbol{E}_-, \quad \Delta\epsilon(x, y, z) = \sum_m \epsilon_m(x, y)e^{i\frac{2\pi m}{\Lambda}z}, \tag{3}$$

with $\boldsymbol{E}_\pm(x, y)$ the electric field distribution of the mode profile.

CMT analysis

By changing coordinates to the following rotating frame:

$$\begin{cases} a_+ = b_+ e^{-iC_{11}z}, \\ a_- = b_- e^{-iC_{22}z}, \end{cases} \tag{4}$$

we obtain the following system:

$$\frac{d}{dz}\begin{bmatrix} b_+(z) \\ b_-(z) \end{bmatrix} = -i \begin{bmatrix} 0 & \delta \\ -\delta^* & 0 \end{bmatrix} \cdot \begin{bmatrix} b_+(z) \\ b_-(z) \end{bmatrix}, \tag{5}$$

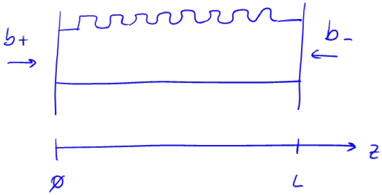
with $\delta = C_{12} e^{i\Delta\beta z}$. The conservation of power in this case read as follows:

$$\frac{d}{dz}\left[|b_+|^2 - |b_-|^2\right] = 0, \tag{6}$$

which yields $|b_+(z)|^2 - |b_-(z)|^2 = \text{const}$.

CMT analysis

The solution to the system of equations read as follows.



We consider an input condition composed of both forward and backward impinging waves:

$$\begin{cases} b_+(0) = A, \\ b_-(L) = B, \end{cases} \tag{7}$$

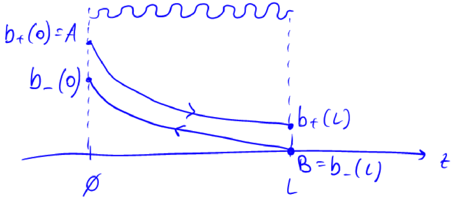
which furnishes the following modal evolution along z:

$$\begin{cases} b_+(z) = e^{i\frac{\Delta\beta}{2}z} \cdot \dfrac{A\left[s\cosh s(L-z)+i\frac{\Delta\beta}{2}\sinh s(L-z)\right]-iB|\delta|e^{i\frac{\Delta\beta}{2}L}\sinh sz}{s\cosh sL+i\frac{\Delta\beta}{2}\sinh sL}, \\ b_-(z) = e^{-i\frac{\Delta\beta}{2}z} \cdot \dfrac{Be^{i\frac{\Delta\beta}{2}L}\left[s\cosh sz+i\frac{\Delta\beta}{2}\sinh sz\right]-iA|\delta|\sinh s(L-z)}{s\cosh sL+i\frac{\Delta\beta}{2}\sinh sL}, \end{cases} \tag{8}$$

with $s^2 = |\delta|^2 - \left(\frac{\Delta\beta}{2}\right)^2$.

CMT analysis

For a complete transfer of energy to occur, we need to observe $b_+(L) \to 0$.



From the solution of the dynamical equation in the previous page, this occurs for:

$$\begin{cases} \Delta\beta = 0, \\ L \to \infty, \end{cases} \tag{9}$$

i.e., for an infinite long structure in phase-matching.

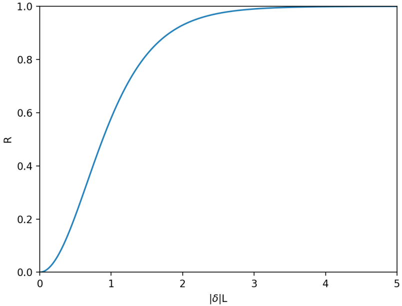
However, while complete transfer is not possible in practice, we observe that the power $|b_+|^2$ decreases exponentially inside the Bragg grating. Hence, even with just a few layers, if the structure is in phase-matching, a large transfer of energy is achieved.

Bragg modal reflectors

The main figure of merit of this interaction is the Reflection coefficient $R = \left|\frac{b_-(0)}{b_+(0)}\right|^2$.

This is calculated when only a forward wave is present, i.e., in the case of $B = 0$. In phase-matching condition $\Delta\beta = 0$, The reflection coefficient is readily evaluated from (8):

$$R = \tanh(|\delta|L)^2 \tag{10}$$

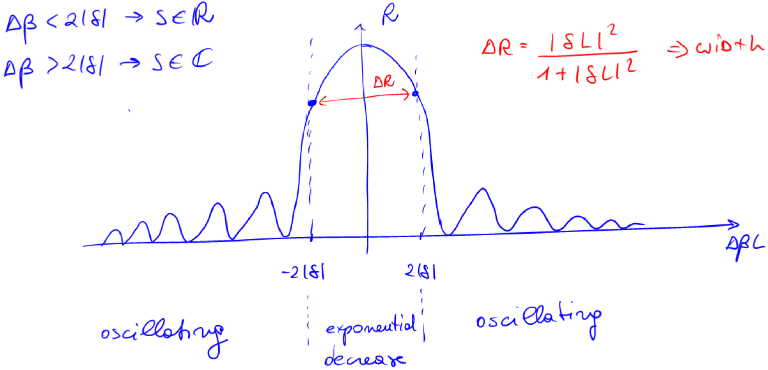


Even with a normalized length $2|\delta|L$, more than 90% conversion of energy can be achieved with a Bragg grating structure.

Bragg modal reflectors

In the general case with mismatch, the reflection coefficient R reads:

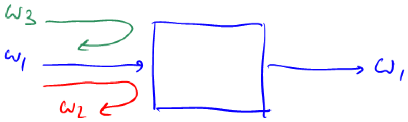
$$R = \left| \frac{|\delta| \sinh(sL)}{s\cosh(sL) + i\frac{\Delta\beta}{2}\sinh(sL)} \right|^2, \tag{11}$$



Outline

Add-drop filter

The mismatch $\Delta\beta = \frac{4\pi}{\lambda} n_{eff} - m\frac{2\pi}{\Lambda}$ is a function of the wavelength $\Delta\beta(\lambda)$. The Bragg reflector therefore acts as a wavelength dependent, add-drop filter:



In this device, all frequencies with $\Delta\beta(\lambda) \in [-2|\delta|, 2|\delta|]$ will undergo exponential suppression inside the Bragg grating, and will be reflected by the structure. This result implies that, for a sufficiently long grating along z, an entire band of frequencies will be forbidden to propagate inside the structure and will be reflected.

Let us investigate the generation of such frequency gap in more detail in the next slides.

Outline

The generation of a Photonic bandgap

The propagation constant β of the forward and backward wave, as seen from (8), is:

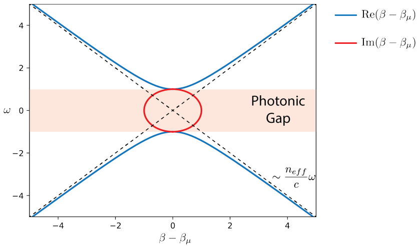
$$\beta = \frac{\Delta\beta}{2} \pm is = \frac{\Delta\beta}{2} \pm i\sqrt{|\delta|^2 - \left(\frac{\Delta\beta}{2}\right)^2} \tag{12}$$

The mismatch term $\frac{\Delta\beta}{2} = \beta_\mu - m\frac{\pi}{\Lambda} = \frac{\omega}{c}n_{eff} - m\frac{\pi}{\Lambda}$ can be rewritten by defining a central frequency ω_0 where $\Delta\beta = 0$, this obtaining $\frac{\Delta\beta}{2} = \frac{n_{eff}}{c}(\omega - \omega_0)$. The total propagation constant then reads:

$$\beta - \beta_\mu = -m\frac{\pi}{\Lambda} \pm i\sqrt{|\delta|^2 - \frac{n_{eff}^2}{c^2}(\omega - \omega_0)^2} \tag{13}$$

All frequencies ω for which (13) contains a nonzero imaginary term are exponentially suppressed, and cannot propagate inside the gratin, generating a gap of forbidden frequencies. Conversely, the ω for which (13) is real can propagate inside the grating, and belong to a band of allowed states.

The generation of a Photonic gap



From the analysis of (13), the Photonic gap region is generated within the frequency range:

$$|\omega-\omega_0| < \frac{c|\delta|}{n_{eff}} \tag{14}$$

For $|\omega| \to \infty$, $\beta-\beta_\mu \sim \pm\frac{n_{eff}}{c}\omega$, and follows the dispersion relation of an unperturbed waveguide. When these two curves intersect, due to the light-matter interaction with the grating, the bands split and a gap of forbidden frequency generates in the dynamics. From (14), the gap width is proportional to $|\delta|$, which in turns is proportional to the refractive index of the perturbation $\Delta\epsilon$. The larger the refractive index, the larger the Photonic gap width. The physical mechanism of gap creation is identical to the band splitting and gap generation in periodic crystalline electronic materials, and as such photonics bandgap media are called Photonics crystals.

Outline

Reference texts

- A. Yariv and P. Ye, *Photonics: optical electronics in modern communication* (Oxford press, 2007), Chap. 12 and Chap. 16